DOCUMENT RESUME

ED 349 184 SE 053 190

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TITLE Preparing Elementary Teachers to Teach Mathematics: A Problem-solving Approach. Final Report. Volume V: Research and Evaluation Component.
INSTITUTION Indiana Univ., Bloomington. Mathematics Education Development Center.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE Jun 92
NOTE 166p.; For volumes I-IV, see SE 053 186-189.
PUB TYPE Reports - Descriptive (141) -- Reports - Research/Technical (143) -- Tests/Evaluation Instruments (160)

EDRS PRICE MF01/PC07 Plus Postage.
DESCRIPTORS Case Studies; College School Cooperation; Elementary Education; Elementary School Mathematics; Elementary School Teachers; Field Experience Programs; Higher Education; Instructional Materials; Mathematics Achievement; Mathematics Education; *Mathematics Instruction; Mathematics Teachers; Mathematics Tests; *Preservice Teacher Education; *Problem Solving; Program Evaluation; Questionnaires; Student Attitudes; Teacher Attitudes; Teaching Methods; *Test Construction
IDENTIFIERS Subject Content Knowledge

ABSTRACT

The "Preparing Elementary Teachers to Teach Mathematics (PETTM) Project" (1988-1991) was a cooperative university/school effort to improve the teaching of mathematics by elementary teachers with its primary focus on improving the university training of preservice teachers (PSTs) in mathematics. This document, the last of the project's 5-volume final report, describes the "Research and Evaluation" component of the project. The first four sections of this volume focus on data relating to the four main project goals. In addition, there is a section on the evaluation instruments that were developed and used in the project. A summary of the project evaluation indicates that the PSTs have formed beliefs that are conducive to teaching mathematics from a problem-solving perspective, that a working relationship with the local school district did take place, and that the PETTM Project did have positive impact on teachers teaching and on students problem-solving abilities and attitudes. Six appendices comprising the bulk of the volume testing program) provide data on mathematical beliefs of PSTs, the Highland Park Teacher Questionnaire and results, profiles of five teachers, ISTEP (Indiana Statewide Testing Program) results, the Problem Solving Tests and results, and student belief interview schedules. (MDH)

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ED349184

Preparing Elementary Teachers to Teach Mathematics:

A Problem-solving Approach

Final Report

National Science Foundation Grant TEI 8751478

Volume V: Research and Evaluation Component

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS: A PROBLEM-SOLVING APPROACH

OVERVIEW OF THE ENTIRE PROJECT

The *Preparing Elementary Teachers to Teach Mathematics Project* (1988-1991) was a cooperative university/school effort to improve the teaching of mathematics by elementary teachers. The primary focus of the project was the improvement of the university training of preservice teachers (PSTs) in mathematics. In the late-1980s, Indiana University had been involved in making plans to institutionalize a new four-year program for elementary education majors in 1989. Two elements of this new program were directly related to the work of the *Preparing Elementary Teachers to Teach Mathematics Project* (*PETTM*). These two elements are:

1. each elementary education major must complete an 18-24 semester credit hour concentration in an approved area (one of those areas being mathematics); and
2. each prospective elementary education major must take part in a three semester sequence of concentrated early field experiences working in a school prior to student teaching.

The goals of the *PETTM* Project included:

1) better preparing all elementary school teachers through a problem-solving focus in newly developed mathematics courses at the university;
2) developing a program of concentration in mathematics consisting of some existing courses and some newly developed courses and experiences to prepare students who wish to make mathematics their field of concentration;
3) developing a close working relationship with the local school district to jointly plan for intensive early field experiences for all prospective teachers;
4) establishing support mechanisms to help practicing teachers in the school district broaden their views of the nature of mathematics and to help them utilize innovative teaching techniques for improving their pupils' mathematical thinking skills and use of problem-

solving strategies. As a result, local teachers should eventually be able to serve as role models in the teaching of mathematics using a problem-solving approach for the prospective teachers who visit their classrooms.

Background and Rationale

A number of national reports published in the mid-1980s called for the improvement of education at all levels. Three reports which attracted national attention were the NCATE Redesign (Gollnick & Kunkel, 1986), the Holmes Group report (1986), and the report of the Carnegie Task Force (1986). Based on the assumption that the quality of education is directly related to quality of teachers, these reports proposed dramatic changes in the practices used in recruiting, preparing, and certifying elementary school teachers. Both the Holmes Group and the Carnegie Task Force reports called for the elimination of undergraduate programs. All of the reports proposed broader and deeper backgrounds in liberal arts. The reports also called for closer ties and working relationships between teacher preparation institutions and school districts.

An elementary education Task Force at Indiana University studied the reports and, while seeing merit in many of their recommendations, declined to join the Holmes group. Although educators at Indiana University certainly agrees with the need for graduate work in education, they also feel that such study is more profitable after some teaching experience has been acquired. Further, the Task Force believed that it is quite possible to train elementary school teachers in a well designed four-year program (cf., Mehlinger, 1986). Thus, in 1989, Indiana University instituted their new four-year elementary education program, a program which incorporated many of the recommendations common to the reports mentioned above. Faculty in Mathematics Education and in Mathematics at Indiana University saw the revision of the teacher education program as a unique opportunity to improve the mathematics preparation of elementary teachers.

Project Description

The *Preparing Elementary Teachers to Teach Mathematics Project* was comprised of four components, all of which were designed to contribute to improving mathematics instruction in elementary schools: the *Content Component*, the *Methods Component*, the *School Component*,

and the *Research and Evaluation Component.*

The *Content Component* involved revision of the university mathematics courses required of PSTs in a fashion that could be exported to other teacher training institutions. Under the new Elementary Education program, prospective elementary teachers would take two mathematics courses: a finite mathematics course (M118—already required of freshmen intending many majors) , and a newly designed mathematics course specifically for education majors (T104). Under project auspices, the former course was restructured and the latter designed completely from scratch, so that problem solving is now a central focus in both. In other words, efforts are now being made to teach university-level mathematics with the same problem-solving approach that the Project would like to see teachers use when they work with children.

The *Methods Component* of the Project had three major goals: 1) to develop well-structured field experiences for both the existing mathematics methods course and a proposed new problem-solving methods course, 2) to develop and institutionalize the new problem-solving methods course (to be required, along with additional mathematics courses, of those students who would elect mathematics as their area of concentration), and 3) to develop a computerized problem-solving data bank to be available for use not only by students enrolled in methods classes and field experiences, but also by prospective teachers in other courses and local teachers in the schools.

An important part of the training of PSTs is experience with classroom teachers and their students. The purpose of the *School Component* of the Project was for Indiana University faculty to work together with faculty from a local elementary school (Dyer/Highland Park) to provide the best possible environment for early field experiences in mathematics for IU students. A full-time resource teacher worked closely with the Dyer/Highland Park School faculty for three years, helping them develop the confidence and expertise necessary to teach mathematics using a problem-solving approach. Five teachers from the school worked for two summers writing books of problem-solving materials that they, and their colleagues, could use with their students. The university-school partnership was a very successful part of the project.

The main purpose of the *Research and Evaluation Component* was to determine the

effectiveness of each of the other components of the project. A long range goal of the entire project was to provide an improved model for training elementary teachers. By finding out which innovations were effective, the project was able to provide a tested plan for training elementary teachers to teach mathematics.

Conclusion

The *Preparing Elementary Teachers to Teach Mathematics Project* was multifaceted, aiming to improve the teaching of mathematics in the elementary schools by encouraging teachers to use a problem-solving approach. Although the major focus of the Project was the design and implementation of a new program of studies for prospective elementary teachers at Indiana University, an essential element in this endeavor was the establishment of Dyer/Highland Park Elementary School as a site for all Project-related field experiences, and the provision of frequent and ongoing inservice work for the Dyer/Highland Park faculty so that school could serve as a model site for the teaching of mathematics with a problem-solving approach.

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS: A PROBLEM-SOLVING APPROACH

Overview of the Research and Evaluation Component

The research and evaluation (R&E) component was somewhat unique in that it had the primary purpose of coordinating research on the project and evaluating the effectiveness of the other components. R&E personnel worked closely with personnel from each of the other components to document the effectiveness of the project as a whole. Originally, it was the intention of the project staff to hire an outside evaluator. Two individuals with sufficient background and skills to assume this position were interviewed and invited to serve as evaluators. Unfortunately, other commitments precluded them from being involved and they declined to accept the invitations. Despite additional efforts, no other candidates were identified who were willing to be evaluators given the resources available for an evaluation. For this reason, project staff, under the direction of the project advisory board, carried out the evaluation effort.

The research and evaluation section of the final report focus on data relating to the four main project goals: teaching with a problem-solving focus, developing a mathematics area of concentration for preservice teachers (PSTs), collaborative work with the local school district, and helping teachers improve mathematical understanding in students. In addition, because instrument development was a major focus of the component, there is a section on the evaluation instruments that were developed and used in the project.

Goal 1: Teaching With a Problem-Solving Focus

The first major goal of the PETTM project was to better prepare all prospective elementary teachers at Indiana University to teach with a problem solving focus. This goal was probably *the* key to the success of the project as a whole so it was included to some extent in the activities of each of the components. As documented in the chapter on the content component, the mathematics course M118 (finite mathematics) was revised to provide elementary education majors with a course in which they would learn to solve mathematical problems as well as participate in a "real" as opposed to elementary education oriented college mathematics. Because the institution of M118 as a requirement for elementary education is quite new, no formal data are available on the effectiveness of the course for elementary education majors. Informal discussions with elementary education students who have completed M118 indicate that some liked the course, some disliked it, but almost all felt they learned a lot of mathematics. As a rough measure of the ability of elementary education majors to complete M118, the grades of 25 elementary education majors were compared with the grades of the entire 250 student population in one section of the course. The mean for elementary education was 2.25 (on a 4 point scale) and the mean for all students was

2.30 indicating that elementary education majors were capable of doing mathematics at the same level as students in other majors.

The second required mathematics content course, T104, was developed as part of the project. The course was quite different from traditional mathematics courses in that small group problem solving was used in place of traditional lecture and discussion. A considerable amount of interview and case study data on the effectiveness of T104 are presented in the content component section of this report. As was the case with M118, most students felt they learned a lot in T104 and that the course helped them to better understand the mathematics they would be teaching to elementary school students.

To supplement the qualitative data reported in the content component of this report, scales to measure the mathematics related beliefs of PSTs were developed by the project. As explained in the instrument development section of this chapter, the scales measure (a) the extent to which an individual believes effort can increase mathematical ability (EFFORT), (b) the extent to which word problems are thought to be an important component of mathematics (WORD PROBLEMS), (c) the extent to which one feels that word problems can be solved using step-by-step procedures (STEPS), (d) the extent to which one feels that understanding concepts is important in mathematics (UNDERSTANDING), (e) confidence in solving difficult mathematics problems (DIFFICULT PROBLEMS), and (f) perceived usefulness of mathematics (USEFULNESS).

The mathematics beliefs scales were administered to PSTs during the first week of T104, again during the first week of E343 (the elementary mathematics methods course), and finally during the last week of E343. As most students took T104 after M118 but one or two semesters before enrolling in E343, changes in scores across these three administrations of the scales represent changes in beliefs during T104 or during E343. Complete data for each administration of the scales is shown in Appendix A. Note that because T104 is a new course, only 40 students had completed both T104 and E343 at the time of this report and thus findings concerning changes in beliefs are tentative at this time. As more students complete the course sequence, it will be possible to generate a more generalizable picture of the extent to which beliefs change during these courses.

Briefly, there were no statistically significant changes on the EFFORT, DIFFICULT PROBLEMS, or USEFULNESS scales between the first and third administrations. Scores improved on the WORD PROBLEMS and STEPS scales, indicating T104 and E343 were effective in convincing students that word problems are important and that many cannot be solved simply by following a series of steps. Although scores on the scales indicated that PSTs had reasonably positive beliefs at the beginning of T104, initial scores on the WORD PROBLEMS and STEPS scales were lower than on the other four. From the first to third administration of the scales, there was a significant decline in the belief that understanding underlying principles is an important part of learning mathematics (UNDERSTANDING). This result is puzzling although it may be due to

a ceiling effect on the first administration of the scales. The belief that one must understand underlying concepts was very strong on the initial administration of the scales. So as not to sound like they were "saying what the evaluators wanted to hear," some students may have been less positive in their responses during the second administration of the UNDERSTANDING scale in E343. A second possibility is that prior to the strong emphasis on teaching for understanding in both T104 and E343, some PSTs may not have really understood what it meant to thoroughly understand a mathematical concept. To many students and teachers, "understand" means memorize a rule better rather than form a deep, conceptual understanding of a procedure. In either case, data from the scales as a group indicate that most students leave E343 with the type of mathematics related beliefs we would like them to have.

Preparing PSTs to teach mathematics was a focus of all of the components. As was outlined in the methods component section of this report, new early field experiences in mathematics were developed as part of the project. The field experiences were based on video-taped sample lessons. A tutoring center and a problem-solving data bank were also developed. As part of the school component, extensive work was done with the teachers at Highland Park Elementary school to make them partners in the education of PSTs. These aspects of the project were designed to meet two or more project goals and will be discussed in more detail in the following pages.

Goal 2: Developing a Mathematics Area of Concentration for PSTs

The second major goal of the PETTM project was to develop a viable mathematics area of concentration for elementary education majors. As outlined in the methods chapter of this report, this area of concentration has been developed. Students wishing to make mathematics their field of specialization have two options for completing 18 semester hours in mathematics beyond the M118 and T104 courses required of all elementary education majors. The first option involves a pre-calculus course, two calculus courses for non-majors, a computer science course, a methods of teaching problem solving course (see the methods component chapter), and a statistics or a Euclidean geometry course. The second option includes the calculus sequence for mathematics majors, the methods of teaching problem solving course, and two other advanced level courses. Although more challenging, the second option provides a student with the mathematics courses necessary for an Indiana "endorsement" to teach middle school mathematics.

To date, the area of concentration program has not been in place long enough for any students to complete it. The methods of teaching problem solving course will be offered for the first time in the fall of 1991 and thus students with a mathematics area of concentration will be student teaching and graduating in the spring of 1992. Students getting an area of concentration in mathematics have been positive about the program although it is too early to have a complete picture of the effectiveness of the program.

Goal 3: Collaboration With The Local School District for Early Field Experiences

The third major goal of the PETTM project was to work with the local school district, and one elementary school in particular, to jointly plan and carry out early field experiences in mathematics. This was the major focus of the school component, and, as explained in the school component chapter of this report, a great deal was accomplished. An overview of those accomplishments will be outlined here. Readers wishing additional detail should consult the chapter describing the school component.

The school selected as the primary collaborator on this project was Dyer Elementary in Bloomington. Dyer was chosen because of the cultural diversity of students at the school and because of the interest of the teachers and principal in joining with project staff in a cooperative venture in mathematics. Note that at the beginning of the third year of the project, students and staff at Dyer moved into a new building called Highland Park Elementary. Although the official name of the school changed, the project continued with the same students and teachers in the new facility.

During the first year of the project, inservice was primarily developed by project staff although by the second year, faculty had considerable input into decision making for inservice and preservice teacher education activities. Funds were obtained through Indiana's Eisenhower (Title II) grant program to offer inservice workshops in mathematics during years 1 and 2. As explained in the school component chapter, the workshops were open to all elementary school teachers in the area although only those from Highland Park were involved in the planning and were expected to attend. Each year, there were six after school workshops, all presented by nationally known experts in elementary mathematics instruction. Followup at Highland Park was the responsibility of the mathematics resource teacher (Jacqueline Gorman) together with other Highland Park faculty. Followup at other schools was much less formal, often involving teams of teachers who had attended the workshops getting together to share ideas. Complete evaluation data of the projects can be found in the final reports of each (Kloosterman, Gorman, Kroll, LeBlanc, Lester, & Shedd, 1989; Kroll et al., 1990). Briefly, participants were quite pleased with the quality of the presentations and with the ability of the speakers to take difficult mathematical concepts (e.g., problem solving, estimation, fractions) and show how to teach those concepts to students. In the words of one teacher, "There were practical, stimulating ideas." Another said she learned "ways to get all children involved, responding, excited about math."

Teachers from Highland Park were also involved in presenting inservice education programs to teachers at other schools in the district or in neighboring districts. In many cases, these teachers also served as "mentors" to those who attended their workshops. The mentoring role involved regular contact with the teachers as well as visits to their classrooms. Highland Park teachers were also involved in the creation of resource notebooks that were disseminated to all elementary

schools in the district. In brief, a great deal was done to form a network of teachers with both the training and expertise to serve as model teachers of mathematics. This network is centered at Highland Park Elementary but includes teachers at all schools in the district and in many schools outside the district. Evaluation data on the effectiveness of the network is limited to anecdotal comments from participants and their principals. These comments have been extremely positive.

As noted, building a network of well trained elementary mathematics teachers is an important goal in and of itself. For the purposes of the PETTM project, however, well qualified teachers are essential for high quality early field experiences in mathematics. The main content and activities for the M201 early field experience at Indiana were developed by PETTM project staff with input and suggestions from teachers, especially those at Highland Park. Teachers were more heavily involved in other aspects of the early field experience. Mathematics fairs were held at Highland Park for two years. Teachers worked with PSTs to plan and help students complete their math fair projects. Mathematics tutoring programs were established at both Highland Park and Grandview elementary schools. Again, teachers were essential in the orderly operation of those programs. Comments from both PSTs and cooperating teachers were very positive for both programs.

Goal 4: Helping Teachers Improve Mathematical Understanding in Students

Obviously, all of the inservice and collaborative activities mentioned as part of goal 3 were designed to help teachers improve their teaching skills. Goal 4 was more specific in that it dealt with helping teachers (1) broaden their views of the nature of mathematics, (2) use innovative teaching techniques to improve pupil's mathematical thinking skills, and (3) do a more extensive job of teaching problem solving strategies. Because inservice efforts have already been discussed, this section of the research and evaluation report will focus on evidence that students at Highland Park were benefiting from the improved teaching skills of their instructors. Because there are a lot of data on teachers and students at Highland Park, this section of the report has been divided into five subsections: summaries of solicited teacher comments, a qualitative study of five teachers, standardized test data on students, problem solving test data on students, and data from belief interviews with students.

Summaries of Solicited Teacher Comments

In May of 1989 and again in May of 1990, all Highland Park teachers were asked to respond to brief questionnaires about the impact of the project on their teaching. Selected teachers were interviewed in May of 1989 and again in May of 1990. Three teachers made a presentation of their reactions to the project to the project advisory board in October of 1990 and all teachers were interviewed a final time in May of 1991. Summaries of teachers' responses to the May 1989 and May 1990 questionnaires can be found in Appendix B. A summary of the comments made during the advisory board meeting is also included. Summaries of the responses to the May 1991

interviews are integral to the discussion of the School Component and thus can be found in the School Component Chapter.

All Highland Park teachers reported having benefited from the project in some way. Although there was considerable variation in the extent to which Highland Park teachers chose to be involved in the project, all were involved to some extent. At the minimal level, every regular education teacher and most of the special education teachers attended at least some of the inservice workshops and made some use of the mathematics resource teacher (Jackie Gorman). All regular education teachers worked with preservice teachers (PSTs) as part of the M201 field experience. Several of the special education teachers were involved with the field experience as well.

The results from the questionnaire given to teachers at the end of the first year of the project (May, 1989) indicated that the project was having a considerable effect, even after one year. Complete questionnaire results are presented in Appendix B. Briefly, the results indicate that almost all regular and special education teachers at Highland Park spend more time developing ideas and less time on drill and practice than they did before the project. There is more use of manipulatives, more problem solving, and an overall change in perceptions of what mathematical ideas need to be taught at the elementary level. A comment made by one teacher in a discussion to followup the questionnaire was that "At the beginning of the year, I had my students write about their feelings toward mathematics. Ninety percent indicated they disliked or even hated it. I repeated the assignment a couple of weeks ago and 90% said they like the subject."

Highland Park teachers were surveyed again at the end of the second year of the project (May, 1990). They were asked to reflect on their teaching practices before the project began, at the end of the first year of the project, and at the end of the second year of the project. The results, reported in detail Appendix B, show that the teachers felt that they were improving each year of the project. No statistical testing of the data was done because of the number of teachers at Highland Park was too small to make such testing appropriate. The teachers did indicate, however, that by the end of the second year of the project, they were teaching a considerable amount of problem solving, they were using manipulatives to develop understanding of concepts, they had become very reflective about their teaching of mathematics, and they were confident that what they were teaching math was appropriate.

One quantitative instrument was used with Highland Park teachers. The teachers were asked to complete the *Indiana Mathematics Belief Scales* (Kloosterman & Stage, 1992) in August of 1988, August of 1989, and again in August of 1990. Means and standard deviations for the teachers who completed the scales can be found in Appendix B. As can be seen from the mean scores, teachers had quite positive beliefs about mathematics throughout the duration of the project. Changes in the makeup of the Highland Park faculty meant that only six teachers completed all three administrations of the scales and thus there was no statistical testing for changes in beliefs.

The final round of teachers comments and reactions to the project was solicited at the end of the third year of the project (May, 1991). Again, there was overwhelming support for the project. Teachers felt that they had undergone fundamental changes in the ways in which they taught mathematics and understood how children learned mathematical concepts. In addition, about half of the teachers who had been involved in the project were involved in inservice projects, conference presentations, or coaching of teachers at other schools. Complete data on the final round of interviews can be found in the School Component.

A Qualitative Study of Five Teachers

Although written comments and interview data can provide considerable insight into how a teacher works with her or his class, observation of a teacher provides the most complete data on how an individual teaches. Five teachers at Highland Park were selected to be observed and interviewed about their teaching practices. Complete reports on each of the five teachers can be found in Appendix C, although a summary of the findings from that appendix is included here.

The five teachers observed were chosen because they represented a cross section of grade levels, and because they agreed to be observed. Interest and involvement in the project was also a criteria for selection because we had little interest in documenting the effectiveness of the project on teachers who had only modest interest in it. Four of the teachers observed had been involved in writing resource notebooks for the project (see the School Component chapter). One was in her second year of teaching when the project began but all others were individuals with substantial teaching experience. The number of observations per teacher varied from two to six although data from all observations were supplemented with interviews and discussions.

Comments made by the five teachers studied, together with the classroom observations, indicated that all five had been affected by the project and were considerably stronger mathematics teachers because of the project. It is interesting to note that each of the teachers found different aspects of the project to be personally interesting and useful. In other words, the varied aspects of the school component were a considerable strength of the project. Almost everything done through the project was valuable, but each teacher ended up using the suggestions made by project staff in their own unique ways.

By the end of the project, all five teachers used manipulatives and were concerned about how well students understood concepts rather than simply getting correct answers. All used a considerable amount of verbal discourse in their classes to help students understand concepts and to check for misunderstandings. Each teacher had students involved in some group work and a variety of problem-solving activities. Among those activities was a "problem of the day" format in which one nonroutine problem was solved each day. Primary grade teachers tended to rely heavily on this while intermediate grade teachers used activities where several days might be spent on the same problem. All teachers expressed confidence that they knew what would help their

students most. Specifically, four of the five expressed little hesitation to forgo the textbook curriculum in favor of supplemental activities that did more to promote broad understandings.

When asked about the aspects of the PETTM project that were most useful, the five teachers mentioned the inservices and manipulative materials provided by the project as important. All five also mentioned that having a mathematics resource teacher *in the school* was very helpful when it came to trying ideas suggested in the inservices. The most consistent concern expressed by the five teachers observed was the lack of time to reflect on what they were teaching to their students. All wished more time was available to talk with other teachers about the methods they were trying and the successes they were having. The teachers who were involved in summer writing of curriculum materials for the project felt that writing was a valuable experience and the summer writing effort provided time for discussion of teaching ideas. Unfortunately, none of the teachers were able to provide insight into how more discussion and reflection time could be found during the school year.

In brief, the five teachers studied were very positive about the project and have become considerably stronger mathematics teachers as a result of it. Much more detail on this part of the evaluation can be found in Appendix C.

Standardized Test Data on Students

Standardized test data are very poor as measures of problem-solving achievement. However, standardized achievement tests were given to all students at Highland Park in March of each year and thus getting standardized achievement data was quite easy. For this reason, standardized test data are included in this report.

As part of the Indiana statewide testing program, students at several grade levels were required to take the *California Achievement Test* (CAT) each spring. In the planning year for this study (1987-88), the statewide program required testing of elementary students in kindergarten, first, second, third, and sixth grade. Kindergarten was dropped from the testing program in the summer of 1988 and thus kindergarten data were collected for only one year. The Monroe County Community School System (MCCSC) required administration of the CAT in grades four and five and thus complete data were available for all Highland Park students in grades 1 through 6 whose parents allowed project staff to see the test scores. These data, consisting of the percentile rank for each student, were collected on the three mathematics portions of the CAT (computation, concepts and applications, total math). Percentile data rather than raw scores were used because the content of the CAT changed at each grade level. This change meant that achievement gains on the test could not be measured directly. However, by using percentile data, it was possible to look at change in performance in relation to a national sample.

Although the PETTM project had a mathematics focus, CAT data from other subjects were also collected to get a better picture of the overall achievement levels of Highland Park students.

Data from March of 1988 were considered baseline data because there had been very few project activities at Highland Park prior to this testing period. In addition to the March 1988 testings, there were testings in March of 1989, 1990, and 1991. A complication in the testing occurred prior to the March 1991 testing when Indiana officials decided to use the *California Test of Basic Skills* (CTBS) in place of the CAT. Schools were provided with 1990 CAT data reanalyzed to be comparable to 1991 CTBS data for comparison purposes. For this project, it was possible to look at gain scores (based on percentile ranks) for 1988-89, 1989-90, and 1988-90. Recalculated 1990 scores were also compared with 1991scores. One must be cautious in evaluating the PETTM project based on gain on standardized test scores given that the project focused on problem solving in much greater depth than either of the tests measured. One must be even more cautious in interpreting the 1990-91 comparisons given that the CAT and CTBS are not equivalent types of tests.

Appendix D gives tables of means, standard deviations, and year-by-year *t*-tests for Highland Park students. Note that descriptive statistics for the same year and grade level vary somewhat by table because they represent only those students who took tests for both years that the table describes. For example, the 1989 first grade mean for the math concepts and applications test shown in Table 1A was 58.6. The 1989 first grade mean for the same test was 62.6 in Table 1B. This discrepancy is due to the fact that only those students who took this test in both 88 and 89 were counted in Table 1A and only those who took it in both 89 and 90 were counted in Table 1B.

Looking through Tables 1 through 7 in Appendix D, one can see that very few of the *t*-tests were statistically significant at the $p<.01$ level. Of the 114 *t*-tests, only 18 (16%) were significant. However, changes on 17 of these tests were negative. That is, on the small number of standardized tests where statistically significant results were found, the changes indicated that students at Highland Park were losing ground in relation to national averages. Looking at the data more closely, it can be seen that most of the losses took place between the testing in March of the first grade year and March of the second grade year. (Seven of the 17 significantly negative tests were for first as compared to second grade and three more were for first as compared to third grade.) In addition, 12 of the 17 significantly negative tests were for mathematics rather than other subjects.

At first glance, the decreases in mathematical performance of Highland Park students on mathematics standardized tests are discouraging. Although the PETTM project was not aimed at improving the skills measured by standardized tests, it was assumed that the project would in no way harm those skills. It is possible that the project did in fact have a negative impact on lower cognitive level skills of students. Other data, however, can be used to argue against such a conclusion.

The strongest argument that students' lower cognitive level mathematics skills were not diminished as a result of the project is the testimony of the students' teachers. As explained earlier in this chapter, teachers had very positive comments about the project and were continually giving examples of how their participation in the project was improving their students' problem solving and computational skills. Another argument is that there were far more non-significant than significant results. Clearly, there is no evidence that the project was uniformly detrimental to students' mathematical skills.

The fact that the majority of the significant decrease took place soon after first grade could be accounted for by the fact that scores in first grade were inappropriately high and thus decrease could be expected. In first grade, the CAT and CTBS tests are simple enough that teachers can easily coach their students on the types of items they will see on the test. The first grade teachers at Highland Park were excellent coaches in this respect and thus probably helped their students get scores that were "too high." The high scores could also be accounted for by the fact that many Highland Park students were in *Head Start* preschool programs. Participation in such programs often gives students an advantage only for the first year or two of school. By second grade, *Head Start* students at Highland Park may have been scoring on tests at a level that was more indicative of their long-term potential.

A final factor that should be considered as encouraging with respect to the standardized test data is that math concepts and applications scores were relatively stable during the last period measured (1990 to 1991). The computation and total math scores went down during fifth grade in 1990-91 (Table 3D) but the math computation score went up in third grade (Table 1D). In other words, by the third year of the project, there was little reason to believe that students' lower-cognitive level skills were being affected by the project.

In brief, standardized achievement tests were administered to all students at Highland Park each year during March. Because the percentile data from the students were easily available, project staff kept track of them from the beginning of the project (1988) through it's completion (1991). Most year-by-year comparisons indicated no significant changes in the skill levels measured by the test indicating that students at Highland Park were progressing at the same rate as their peers in other schools. Where there were changes in scores, performance of students dropped in relation to national averages. However, a close look at these data indicate little cause for concern about the lower-cognitive level skills the standardized tests measure.

Problem-solving Skills

Because standardized tests are not good measures of problem-solving ability, and because no tests were available to measure the type of problem-solving being stressed in the PETTM project, PETTM staff decided to develop a brief problem-solving test that could be used to gather data on Highland Park students' problem-solving skills. The major constraints on the development of this

test were that the amount of time needed to take it had to be kept to 30 minutes or less and, lacking resources for specialists to administer the test, it had to be something that could be done in a classroom situation under the direction of a single adult.

The first administration of the problem solving test took place in May of 1989. The test was actually six different tests, one for each grade level. Each grade-level test had five items. For grades 1 through 3, there was a simple translation problem, a simple translation problem with extraneous data, a "make a list" process problem, a "guess and check" process problem, and a skill exercise. (See Charles & Lester [1982] for a description of problem types.) For grade 4 through 6, the problem types were the same except that the problem with extraneous data was a multi-step rather than a simple translation problem.

The test was administered again in May of 1990 and 1991 with minor modification. The modification involved adding a sixth problem (a logical thinking process problem) and breaking the test into two parts of three problems each. Several of the problems were rewritten to avoid ambiguous interpretations. The final version of the test and scoring guide are provided in Appendix E.

Tables one through six in Appendix E are descriptive statistics on the test. Although the test was administered for three years, comparison of student progress from year to year is not appropriate. As previously explained, it was possible to compare achievement test data from year to year because the data were in relation to national norms. Because the items on the problem solving test were different for each year, and because there are no test data on a larger population, gain scores cannot be computed and improvement in problem solving cannot be measured by this test.

The results of the problem-solving test (Appendix E) are very encouraging. The first item at each grade level was a simple translation problem of the type often seen in textbooks. Full credit on the problem was 2 points, a score that many children achieved. The second problem included extraneous information and was more difficult than the first. As expected, scores on this problem were lower but still high enough to indicate many students were able to do it. Students did well on the skill exercise (Part 2, # 1). Some students had trouble with the process problems (Part 1, #3; and Part 2, #2 and #3) although a number did quite well on these challenging items. In general, the problem solving test results indicate that a majority of the students at Highland Park were learning to be mathematical problem solvers.

Students' Beliefs

The final technique to measure of the effectiveness of the PETTM project in helping teachers improve their ability to teach mathematical thinking skills was to interview students about their beliefs about mathematics and themselves as learners of mathematics. A belief interview instrument was piloted with students in first, third, and fifth grade in May of 1989. Based on the

results of the pilot, the instrument was revised and used with about 10 students per grade level for each grade (1-6) in May of 1990. The complete instrument is shown in Appendix F.

Overall, the findings of the belief interviews were quite encouraging. Highland Park students generally enjoyed mathematics and were confident of their ability to do math. As part of the interview procedure, students were asked to solve problems and use manipulatives. Although some students had considerable difficulty with some of the problems, many had used fairly sophisticated thinking to answer the questions we posed. Trends that emerged from the May 1990 interviews included the following: (a) students in the upper elementary grades were able to articulate beliefs more clearly than students in the lower grades, (b) most students believed that anyone who tried could learn mathematics, (c) there seemed to be no relation between parental support and mathematics achievement, (d) students with moderate achievement were just as confident of their abilities as high achievers, and (e) students who enjoyed mathematics were also confident of their ability in mathematics. A more thorough description of the interview procedure and results can be found in Kloosterman and Cougan (1991).

In an effort to further understand the project's affects on students' beliefs, the students who were interviewed in May of 1990 were interviewed again in May of 1991. Some of the questions asked in 1990 were repeated in these interviews and several questions designed to look at changes in beliefs were added (see Appendix F). Preliminary analyses of the 1991 data indicate that most students continued to be confident of their mathematical ability and had considerably improved ability to solve problems. Complete analysis of the May 1991 data should be available late in 1991.

Instrument Development

Several instruments were developed for use in the PETTM project. The results of these instruments have already been discussed but additional discussion of the instruments will help to put the results in a more meaningful context. Instruments developed were: (a) the *Indiana Mathematics Belief Scales* , (b) Highland Park student problem-solving tests, (c) belief interview instruments for Highland Park students, and (d) achievement indicators for preservice teachers enrolled in the new mathematics content course (T104).

Indiana Mathematics Belief Scales

As explained in the discussion of the first project goal, the *Indiana Mathematics Belief Scales* (IMBS) were developed to measure the effectiveness of the project for helping PSTs form beliefs about mathematics which are in accord with those mentioned in the NCTM *Curriculum and Evaluation Standards* (1989). The scales measure (a) the extent to which an individual believes effort can increase mathematical ability (EFFORT), (b) the extent to which word problems are thought to be an important component of mathematics (WORD PROBLEMS), (c) the extent to which one feels that word problems can be solved using step-by-step procedures (STEPS), (d) the

extent to which one feels that understanding concepts is important in mathematics (UNDERSTANDING), (e) confidence in solving difficult mathematics problems (DIFFICULT PROBLEMS), and (f) perceived usefulness of mathematics (USEFULNESS). These beliefs were selected because of the connection between them and motivation to study mathematics. Specifically, students who feel that effort will increase their ability are more highly motivated than students who feel that effort is not related to achievement. Students who feel word problems are important will be motivated to learn to solve such problems. Students who feel all problems can be reduced to a prescribed series of steps often put their energy into reducing a problem to small, simple minded steps rather than using more global heuristics. Students who feel that understanding mathematics is important spend time trying to understand concepts rather than just trying to get a correct answer. Students who are confident that they can solve difficult problems are not afraid to attack such problems. Finally, students who see mathematics as useful are more likely to be motivated than students who feel knowing mathematics will be of little value to them later in life. See Kloosterman and Stage (1989, 1992) for the complete IMBS and a more thorough discussion their development.

Elementary School Problem Solving

As noted, a problem-solving test was developed for use with students at Highland Park Elementary School. Part of the inservice program at Highland Park involved discussion and use of the *Problem-Solving Experiences in Mathematics* series published by Addison-Wesley. The test was modeled on the types of problems discussed in that series. As noted in the discussion of test results, the final version of the test was actually six different pairs of tests as the six items for each grade level were administered in two groups of three problems each. Students in grades 1 through 3 were given a simple translation problem, a simple translation problem with extraneous data, a "make a list" process problem, a skill exercise in which one only had to circle the necessary information, a "guess and check" process problem, and a "logical reasoning" process problem. (See Charles & Lester [1982] for a description of problem types.) For grade 4 through 6, the distribution of problems was the same except that the problem with extraneous data was a multi-step rather than simple translation problem.

A scoring system for the tests was developed using a focused holistic system (see Charles, Lester, and O'Daffer, 1987). Each problem was assigned one to three points, depending on complexity of the problem. Given the diversity of the problems on the test, there was no overall test score.

To assure validity in the scoring system, two scorers rated the May 1989 tests separately. The correlation between the scores of the two raters varied with problem number and grade level but was greater than .8 on 86% of the items and greater than .9 on 67% of the items. The two scorers discussed all items where there was disagreement with PETTM project staff so that a

consistent scoring system was applied in all cases. As a result of the discussions about scoring, the two scorers of the May 1990 and May 1991 were in complete agreement on most items. In the handful of cases where there was a discrepancy, discussion of the students' work on those items resulted in a consistent scoring system at all grade levels.

In retrospect, there were two significant problems with using a focused holistic scoring system on a research based project. First, students' writing was often hard to read, particularly at the primary grade levels. Scorers had difficulty in trying to decide what number was actually written down or which of several numbers shown was intended as the final answer. Second, many students did not show their work. This was particularly true in the first administration of the test. At the time the PETTM project began, many teachers were telling students to erase mistakes on homework and quizzes. Not surprisingly, they did the same thing on our problem solving test! Although many students appeared to have been close to a correct answer, they then erased all their work making awarding of partial credit difficult. It should be noted that the limitations of a focused holistic scoring system may be unique to research ventures. Individual teachers can probably use a focused holistic system effectively because (1) they can usually decipher their students' writing, and (2) they can teach children to show their work rather than erasing anything that might be considered incorrect. The final version of the problem-solving test and scoring guide are provided in Appendix E.

Testing Mathematical Knowledge of Preservice Teachers

As part of the content component of the PETTM project, a variety of alternative assessment measures for the T104 course were developed. These measures, which include group and individual testing of mathematical knowledge, are described in the evaluation section of the content component report. Actual assessment tasks for preservice teachers (PSTs) and description of a scoring system for those tasks are included in an appendix to the content component report.

Belief Interview Instruments

The belief interview instruments previously mentioned were developed specifically for the PETTM project because no adequate instruments were available. The instruments (see Appendix F) were developed to measure specific beliefs and should be of value to researchers with similar interests.

Initial development of the belief interview instruments took place in the spring of 1989 and focused on discussion of what beliefs PETTM project staff hoped students would acquire. Some of the beliefs were ones that could be discussed directly. For example, we felt it was appropriate to ask students how they thought mathematics could be used. Some beliefs, however, were more subtle. Although we asked students directly about their confidence in solving problems, we also observed their behavior when solving problems to see if they appeared to be confident in what they were doing. To determine what children viewed as mathematical as opposed to non-mathematical,

we developed a card sort of math problems. As children sorted the problems into "math" and "not math" piles, we asked them why they categorized each problem as they did.

Our original thinking on the interview questions was that the type of questions we needed to ask of young children was different than the type of questions we could ask of children in the upper elementary grades. After the pilot interviews in 1989, we found that some children were much better at expressing themselves than others. However, there was not that much age-related variation in the type of questions that students were able to answer. Young children who had strong verbal skills could answer any of the questions we asked. Young children with weak verbal skills had trouble with many of the questions. The same was true to a lesser extent for older students. Based on this experience, we decided to keep the main thrust of the questions the same for all ages. We did, however, make the problems children were asked to solve during the interview more complex at higher grade levels.

The interview procedure piloted in May of 1989 was used with about ten children at each grade level (K-6) in May of 1990. After analyzing those results, a followup interview was devised for use with the same children in May of 1991. The followup interview (1991) differed from the original interview (1990) in several key ways. First, children were usually interviewed in pairs on the assumption that discussion of ideas with a peer would make it easier for the less verbal children to express their thoughts and beliefs. Second, children were no longer asked to solve problems during the interview. This change was made because the problem solving portion of the interview was quite time consuming and, although it helped us understand children's problem solving skills, it was not very useful in determining beliefs. Third, several questions were added to the interview to get children to focus on how their beliefs had changed in a year's time. The complete May 1990 and May 1991 interview instruments are included in Appendix F.

Summary

The research and evaluation component of the PETTM project was integrated into each of the other components as a means of showing the effectiveness of each component and the project as a whole. Some parts of the project produced materials which were useful for documenting the effectiveness of the project. To supplement "product" evidence, specific research initiatives were also undertaken.

The first goal of the PETTM project was to better prepare elementary teachers in mathematics content. To accomplish this goal, the content requirements for elementary education majors were changed. All were required to take M118, finite mathematics. To accommodate elementary education majors, M118 was revised to make it more liberal arts and less business oriented. All elementary education majors were also required to take T104, a new course designed to teach mathematics *via* problem solving. Because changes in course requirements are relatively recent,

there are no evaluation data on the long term effects and implications of the changes. The data that are available indicate that elementary education majors are becoming at least as mathematically literate as students in many of the other non-mathematically-oriented majors at Indiana University. Students in the T104/E343 (elementary math methods) sequence were also asked to complete belief scales at the beginning of T104, at the beginning of E343, and at the end of E343. Although a relatively small number of students have completed this sequence, the data from the surveys indicates that preservice teachers (PSTs) have formed beliefs that are conducive to teaching mathematics from a problem-solving perspective.

The second goal of the PETTM project was to develop a program of concentration in mathematics for preservice elementary teachers. The program is currently in place. However, there are few evaluation data on the program because there are no students who have had time to complete it.

PETTM's third goal was to develop a close working relationship with the local school district to jointly plan for early field experiences in mathematics. At the time of this writing, such a relationship is certainly in place. Although logistical constraints limit participation in the early field experience to two schools, those schools are heavily involved. In addition, inservice and cooperative ventures have taken place in most other elementary schools in the district. Of considerable importance is the fact that much of the inservice to schools is being provided by teachers who have been involved in the PETTM project rather than only by project staff.

The fourth goal of the PETTM project was to establish mechanisms to help practicing teachers broaden their views of mathematics and improve their instruction on thinking skills and problem solving. To achieve this goal, PETTM staff worked closely with teachers at Highland Park Elementary school (previously known as Dyer Elementary.) A variety of measures of the success of this portion of the project are available. First, Highland Park teachers were asked to comment on how the project affected them. All were positive about the experience although the amount of impact varied with the interest of the individual teacher. All felt the project had benefited their students. Second, five Highland Park teachers were participants in a qualitative study of teaching. These teachers were highly impacted by the project and have all made significant improvements in their ability to teach problem solving. Third, standardized test scores were collected for students at Highland Park. Although the problem-solving emphasis of the project did not appear to cause improvement in the basic skills measured by the test, it was probably at least as effective in building those skills as a more traditional teaching approach. Fourth, students at Highland Park were asked to complete a problem solving test each May. The results from those tests indicate that many students are becoming good problem solvers. Finally, interviews with Highland Park students about their mathematical beliefs indicate that most have very positive attitudes about mathematics and have the type of beliefs that are associated with high motivation in

the subject.

In the original proposal for the PETTM project, instrument development was mentioned as a key to evaluation of the project. For this reason, instrument development has also been discussed. Briefly, instruments developed include belief scales, elementary-level problem solving tests, achievement tests for preservice teachers, and belief interview instruments for use with elementary school students. All of these instruments have been used in the project and are either included in this report or are readily available through other sources.

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PREPARING ELEMENTARY TEACHERS TO TEACH MATHEMATICS: A PROBLEM-SOLVING APPROACH

RESEARCH AND EVALUATION COMPONENT PUBLICATIONS AND PRESENTATIONS

Publications Related to the Research and Evaluation Component

Kloosterman, Peter (1992). Non-routine word problems: One part of a problem-solving program in the elementary school. *School Science and Mathematics*, *92*, 31-37.

Kloosterman, Peter, & Cougan, Monica (in press). Students' beliefs about learning elementary school mathematics. *Elementary School Journal.*

Kloosterman, Peter, & Stage, Frances K. (1992). Measuring beliefs about mathematical problem solving. *School Science and Mathematics*, *92*, 109-115.

Presentations Related to the Research and Evaluation Component

Kloosterman, Peter (October, 1990). Are story problems a curse of Western culture? A look at students' beliefs about mathematics. *Regional meeting of the National Council of Teachers of Mathematics*, Madison, Wisconsin.

Kloosterman, Peter, & Cougan, Monica (April, 1991). Students' beliefs about learning elementary school mathematics. *Paper presented at the Annual Meeting of the American Educational Research Association*, Chicago, Illinois.

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Appendix A
Mathematical Beliefs of Preservice Teachers

As part of the research and evaluation component of the PETTM project, preservice teachers (PSTs) at Indiana University were asked to complete the *Indiana Mathematics Belief Scales* (IMBS). The scales, which measure beliefs about learning and knowing mathematics, were developed for the project. On the assumption that the new mathematics course (T104) and the revised field experience and methods courses (M201/E343) would influence beliefs, the scales were administered to PSTs at the beginning of T104, at the beginning of M201/E343, and again at the end of M201/E343. As T104 was intended to be taken one to two semesters before M201/E343, this sequence provided a picture of beliefs at three points in time.

The *Indiana Mathematics Belief Scales* consist of five, 6-item scales that measure beliefs related to learning mathematics. The beliefs are as follows.

- Effort Can Increase Mathematical Ability (EFFORT) measures the extent to which effort can make one "smarter" in mathematics. An assumption behind the development of this scale is that students who think they will always be "dumb" in mathematics have little motivation to study the subject.
- Word Problems are Important in Mathematics (WORD PROBLEMS) measures the extent to which an individual feels that word problems, as opposed to routine computations, are an important part of mathematics. Solving non-routine word problems was an important part of T104 and knowing how to teach students to solve such problems was an important part of E343. Thus, this scale was used to see if PSTs agreed with the PETTM philosophy that mathematics is more than computation.
- There are Word Problems That Cannot be Solved with Simple, Step-by-Step Procedures (STEPS) measures agreement with the notion that "real" mathematics involves more than memorized procedures. Again, it was expected that PSTs would agree with this idea once they had completed the T104/M201/E343 sequence.
- Understanding Concepts is Important in Mathematics (UNDERSTANDING) was designed to measure the extent to which an individual felt understanding concepts, as opposed to simply memorizing algorithms, was an important part of mathematics.
- I Can Solve Time-Consuming Mathematics Problems (DIFFICULT PROBLEMS) measured self-confidence in doing multi-step and other non-routine mathematical problems. The scale is similar to traditional self-confidence scales except that it deals specifically with time-consuming or non-routine problems.

A sixth scale, adapted from previous work, was also used.

- Mathematics is Useful (USEFULNESS) measures the extent to which an individual sees

mathematics as useful in everyday life. The scale was used because of research findings that students who see mathematics as useful are more likely to try to learn the subject than those who do not.

All scales, along with a detailed description of their development, can be found in Appendix ?.

As noted, the *Indiana Mathematics Belief Scales* were administered at the beginning of T104, at the beginning of M201/E343, and again at the end of M201/E343. Because T104 was a new course, only a small number of students had taken both T104 and M201/E343, making analysis of changes in beliefs during T104 difficult. (We will continue to use the scales and thus expect to have much better longitudinal data in the future.) Because there was no reason to expect that students beliefs would vary with the semester in which they enrolled in T104 or M201/E343, data were pooled across semesters. Table 1 gives means and standard deviations of all students who completed the scales at the beginning of T104, at the beginning of M201/E343, or at the end of M201/E343.

As can be seen in Table 1, beliefs of PSTs were neutral to positive on all of the scales. Because there were six items per scale, a score of 18 represented a response of "undecided" and a score of 24 represented a response of "agree." On the importance of word problems and the mathematics as a series of steps scales, PSTs were generally neutral at the beginning of T104 although group means increased somewhat by the end of M201/E343. Respondents rated understanding mathematical ideas as important and mathematics as useful at all administrations of the scales. Note that the number of respondents at the beginning of M201/E343 was considerably larger than the number at the end of M201/E343 because the IMBS was not administered at the end of one semester.

Comparisons between scores at each of the three administrations of the scales were done only for those students who completed more than one administration of the scales. Tables 2, 3 and 4 show paired-sample *t*-tests comparing scores on various administrations of the IMBS. As is evident in all four tables, PSTs beliefs were relatively stable at different points in time. There were no significant differences between the beginning of T104 and the beginning of M201/E343 (Table 2) although the number of individuals who were tested at both of these points in time was fairly small (about 44). Between the beginning and end of M201/E343, there were significant differences on two scales, STEPS and UNDERSTANDING. The significant gain on the STEPS scale suggests that M201 and E343 helped PSTs to see that stepwise procedures have limitations when it comes to solving non-routine mathematics problems. The significant decline on the UNDERSTANDING scale is hard to interpret. The decline, although significant, was quite small. In addition, it should be noted that the decline could be due to a ceiling effect in the scale as the mean scores on both the pre and post administrations were over 25 on a scale with a maximum of 30.

Table 4 shows differences from the beginning of T104 until the end of M201/E343. Again, the number of PSTs tested at these two points in time is quite small and thus data should be interpreted cautiously. As shown in Table 4, there were significant gains on the WORD PROBLEMS and STEPS scales and a significant decline on the UNDERSTANDING scale. The importance of word problems scale is the only one that was significant in Table 4 that was not significant in Table 3. A possible explanation for this is that the heavy emphasis on solving word problems in T104 combined with the heavy emphasis on how to teach children to solve word problems in E343 was particularly effective in changing PST's beliefs about word problems.

Overall, Tables 1 through 4 indicate that PST's beliefs about learning and doing mathematics are fairly positive and relatively stable. By the end of the spring semester of 1992, we should have complete data on over 300 PSTs. These larger numbers will findings easier to accept. It would be interesting to look at beliefs of PSTs before they enroll in their first university mathematics course, finite mathematics (M118). This course is different from most high school mathematics in that it has a heavy emphasis on solving non-routine problems. Because of this emphasis, the course could have a measurable impact on beliefs. Unfortunately, the logistical problems of identifying and testing elementary education majors prior to enrollment in this course make it unlikely that we will be able to look at beliefs of PSTs at the beginning of their college experience.

Table 1.
Means and Standard Deviations for Each Administration of the IMBS.

	Beginning of T104 (264≤*N*≤269)		Beginning of M201/E343 (411≤*N*≤416)		End of M201/E343 (247≤*N*≤262)	
Scale	Mean	(SD)	Mean	(SD)	Mean	(SD)
EFFORT	22.04	(3.72)	21.88	(3.80)	21.26	(3.85)
WORD PROBLEMS	18.83	(3.18)	20.23	(3.19)	21.10	(3.01)
STEPS	18.11	(3.32)	18.34	(3.22)	19.97	(3.38)
UNDERSTANDING	25.25	(2.77)	25.73	(2.60)	25.18	(2.78)
DIFFICULT PROBLEMS	20.86	(4.02)	21.41	(3.87)	21.04	(4.14)
USEFULNESS	23.68	(3.19)	24.70	(2.67)	24.48	(2.66)

*$p<.01$

Table 2.
Means, Standard Deviations, and Paired Sample t -tests for Differences in IMBS Scores Between the Beginning of T104 and the Beginning of M201/E343.

		Beginning of T104		Beginning of M201/E343		
Scale	*N*	Mean	(SD)	Mean	(SD)	*t*
EFFORT	44	21.25	(4.27)	21.14	(3.48)	.18
WORD PROBLEMS	44	19.42	(3.71)	20.19	(3.40)	1.99
STEPS	44	18.50	(3.52)	18.57	(3.22)	.14
UNDERSTANDING	44	25.59	(2.64)	25.34	(2.35)	.70
DIFFICULT PROBLEMS	44	22.05	(3.78)	21.50	(3.61)	1.43
USEFULNESS	42	23.83	(3.51)	24.07	(3.01)	.58

*$p<.01$

Table 3.
Means, Standard Deviations, and Paired Sample t-tests for Differences in IMBS Scores Between the Beginning of M201/E343 and the End of M201/E343.

		Beginning of M201/E343		End of M201/E343		
Scale	*N*	Mean	(SD)	Mean	(SD)	*t*
EFFORT	213	21.83	(3.76)	21.32	(3.82)	2.15
WORD PROBLEMS	215	20.66	(3.24)	21.13	(2.99)	2.25
STEPS	227	18.61	(3.37)	19.99	(3.35)	5.65*
UNDERSTANDING	218	25.72	(2.64)	25.25	(2.74)	2.77*
DIFFICULT PROBLEMS	218	21.18	(4.07)	21.07	(4.12)	.48
USEFULNESS	217	24.68	(2.72)	24.46	(2.68)	1.25

*$p<.01$

Table 4.
Means, Standard Deviations, and Paired Sample t-tests for Differences in IMBS Scores Between the Beginning of T104 and the End of M201/E343.

		Beginning of T104		End of M201/E343		
Scale	*N*	Mean	(SD)	Mean	(SD)	*t*
EFFORT	38	21.71	3.89	22.08	3.80	.52
WORD PROBLEMS	38	19.18	3.72	21.11	2.98	4.13*
STEPS	40	18.40	3.40	20.33	3.80	2.78*
UNDERSTANDING	37	25.46	2.82	24.32	2.86	3.23*
DIFFICULT PROBLEMS	38	21.87	3.80	21.76	4.06	.24
USEFULNESS	36	24.25	3.88	24.53	2.64	.57

*$p<.01$

Appendix B: Questionnaire Data From Highland Park Teachers

Dyer (Highland Park) Faculty Year-end Questionnaire, May 1989

Summary of Results from 14 Regular Education and 5 Special Education Teachers

(Numbers below each choice indicate the number of teachers circling that choice. Teachers were also invited to provide comments. Few did provide comments but summaries of comments have been provided on the items where there were comments.)

1. As a result of the mathematics project, I spend more time stressing understanding of mathematical ideas in my class.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
5	14	0	0	0

2. As a result of the mathematics project, I feel I have a better understanding of how children can be taught mathematical problem solving.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
7	11	1	0	0

3. As a result of the mathematics project, I am including more problem solving in my mathematics lessons.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
8	11	0	0	0

4. I think the mathematics project has made me a better teacher.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
5	13	1	0	0

5. As a result of the mathematics project, I have my students work in cooperative groups more often.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
2	11	5	1	0

6. As a result of the mathematics project, I use more manipulative materials.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
7	9	2	1	0

7. As a result of the mathematics project, I stress estimation in my class more than I did before.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
1	9	6	3	0

comments: two of the three teachers who disagreed said they planned to stress estimation in the following school year

8. As a result of the mathematics project, I spend more time thinking about the mathematics lessons I teach.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
4	13	2	0	0

9. The mathematics project has changed my outlook on the purpose of teaching mathematics in the elementary school.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
5	12	1	1	0

comment: the individual who disagreed said she already viewed mathematics in the way project staff were hoping she would

10. I have utilized Jackie Gorman for ideas and ways to better teach mathematics.

Strongly Agree	Agree	Uncertain	Disagree	Strongly Disagree
6	10	2	1	0

comments: a number of teachers commented on how important it was to have Jackie available for consultation

Please describe one or two areas of your teaching where you feel the mathematics project has had the greatest effect.

Representative Comments:

- I have come to realize, even more so, that sometimes the process is more important that the final product. *(special education)*
- We talk about story problem contents before working on them. *(regular education)*
- The whole attitude toward mathematics -- my kids hated it, now they love it. *(regular education)*
- I am going to revamp my program over the summer and make major changes. I will be using the ideas I learned in the workshops. I'm excited about this! *(special education)*
- It has encouraged and introduced a more creative and interesting approach to the teaching of mathematics (less book oriented) and exposed me to the huge number of resources available. *(regular education)*
- I have gone way beyond just using the math book for math. The math book is used as just one resource. *(regular education)*
- I had to adapt ideas for use in special education but it was easily done! *(special education)*
- I use more manipulatives because they are available. *(special education)*
- I'm not as heck-bent on long and arduous practice as I once was accused of being. *(regular education)*
- I use more manipulatives and get children to talk about what they have done. *(regular education)*

Highland Park (Dyer) Faculty Year-end Questionnaire, May 1990

Summary of Results

In May, 1990, 10 Regular Education and 3 Special Education Teachers Completed a survey of the effects of the project on their teaching. Not all teachers answered all questions. For example, one teacher was on leave for the 1989-90 school year and thus did not complete the survey questions for that year.

Teachers were asked to circle a number from one to five for each of the periods "Before Project (prior to 1988)" "Last Year (1988-89)," and "This Year (1989-90)." The numbers from one to five were to represent "1 = not at all," "2 = a little, but not very much," "3 = some," "4 = a fair amount," and "5 = a lot." The questions were as follows:

1. I spent class time stressing understanding of mathematical ideas.
2. I felt I knew how to teach mathematical problem solving to children.
3. I included problem solving in my mathematics lessons.
4. The mathematics project helped me to become a better teacher.
5. I had my students work in cooperative groups.
6. I used manipulative materials to teach mathematics.
7. I stressed estimation in my class.
8. I spent a lot of time thinking about the mathematics lessons I taught.
9. The mathematics project changed my outlook on the purpose of teaching mathematics in the elementary school.
10. I utilized Jackie Gorman for ideas and ways to better teach mathematics.

Table of Results for May 1990 Highland Park Faculty Year-end Questionnaire.

Question Number	Before Project		Last Year		This Year	
	n	*mean (s.d.)*	*n*	*mean (s.d)*	*n*	*mean (s.d)*
1	11	3.0 (1.0)	11	3.8 (0.8)	11	4.4 (0.6)
2	12	2.5 (1.2)	12	3.0 (1.2)	13	4.2 (0.6)
3	12	2.3 (1.1)	12	3.3 (0.9)	13	4.2 (0.9)
4			12	3.7 (0.9)	13	4.2 (0.7)
5	12	2.5 (1.3)	12	3.3 (1.3)	13	3.5 (1.1)
6	12	2.2 (1.0)	12	3.4 (1.0)	13	4.0 (0.6)
7	12	1.7 (0.8)	12	2.8 (1.1)	13	3.3 (1.1)
8	12	2.7 (1.2)	12	3.5 (1.0)	13	4.3 (0.6)
9			12	3.6 (1.0)	13	4.1 (0.9)
10			12	3.4 (1.7)	13	3.2 (1.2)

In addition to rating the ten items just discussed, teachers were asked three open ended questions. Following is a representative sample of the results.

Please describe one or two areas of your teaching where you feel the mathematics project has had the greatest effect.

- Questioning, making children explain, and allowing more collaboration.
- Realizing that application of mathematical concepts is necessary to successfully teach mathematics and for students to understand its purpose.
- I have more confidence in the professional decisions I make.
- I'm using more collaborative grouping.
- I'm using more manipulatives.
- I teach the actual strategies in problem solving (i.e., guess & check, listing, etc.) so students are familiar with several ways to work problems.
- I enjoy having the children do group projects.
- I stress estimation much more.
- I'm getting students to reduce the urge to get to the answer in exchange for knowledge about how to get to that point.
- It's ok to use calculators.

Has your mathematics curriculum (the topics you teach or the order you teach them) changed as a result of the mathematics project? (Please explain.)

- I have changed the emphasis of my lessons. I teach almost half problem solving and half computation. I am much more apt to use manipulatives. Each year I try to do more.
- I'm less likely to go through the book page by page. I have more of a tendency to skip around to different units and use problem solving throughout.
- I do lots more problem solving.
- Not really. I think not being here last year made it harder for me to make many changes. I really felt like I missed a lot.
- We do mental math, work in groups, and use manipulatives.
- I teach addition, including regrouping first. Then I teach subtraction with borrowing. This seems to follow a more logical route.
- This year I taught graphing first, did <u>not</u> teach long division to my low group, and tried to expose all ability groups to geometry, fractions, and problem solving.
- I incorporated more project problems.
- My math curriculum is integrated more and not broken into sections, every page in the book is not done in order.

Do you think parents' perceptions of mathematics have changed as a result of the mathematics project (through children's comments at home, the math fair, the math newsletter, etc.)?

Seven teachers said at least some parents have come to see math as more than computation, valuable and important. Five were unsure and one did not respond.

Summary of Comments Made by Three Highland Park Teachers at the Meeting of PETTM Advisory Board October 19, 1991

Mrs. C is a first grade teacher, Mrs. B is a third grade teacher, and Mrs. M is a fifth grade teacher. All three met with the project advisory board to discuss their experiences with the project. Following is a summary of the major questions asked by Board members and project staff, along with responses from the teachers.

Q. *Can you give us some idea of what affect this project has had on you and your students?*

Mrs. C. I now see math as much broader than I used to. I used to have one math period, now math is part of everything.

Mrs. B. My students are willing to try, they are not afraid of story problems. *All* of them can do the ones in the book now that they have been in the program for more than two years.

Mrs. C. My students are much better at discussing things because they are used to discussing math.

Mrs. M. I used to think only "smart" students could do problem solving. I now know that they all can do it. I also do a lot more with integrating mathematics into everything I do in the classroom.

Q. *What is the impact of the project on routine mathematical skills?*

All Students are doing just fine on routine skills.

Mrs. M. Students have a much better understanding of routine skills.

Mrs. B. We spend a lot of time discussing problems of the day, this helps routine skills and problem solving skills.

Q. *Is the project spreading beyond Highland Park?*

Mrs. B. All three of us have been doing workshops for other teachers and are now coaching teachers at other schools.

Mrs. M. Some of the teachers I am coaching are able to do even more problem solving than I can because they have more academically oriented students.

Mrs. C. I have organized a primary grade "user's group" that gets together to discuss teaching math and using project developed materials.

Mrs. B. I have a novice group of teachers from other schools. We are talking some now but I plan to have workshops for them in the coming summer.

Q. *What advice do you have for other schools who want to improve their mathematics teaching?*

Mrs. B. It takes time to learn to teach math well. You can't do it overnight.

Mrs. M. The university should provide support but the networks/user groups of teachers are essential, both within schools and across schools.

Mrs. M. When giving workshops for other teachers, I start by giving participants reasons why they should do things differently.

Mrs. B. When doing two day workshops, we inundated participants with examples of what to do, we put them in lots of problem solving situations.

Mrs. M. Teachers are as afraid of problem solving as students are. It was good for them to work with their peers to learn to solve problems.

Mrs. C. I tell teachers that in my class, solving the problem of the day is the "ticket to lunch." Students can't get in the lunch line until they have answered the problem and put their work in the appropriate box.

Mrs. M. When I'm coaching other teachers, I ask how their lessons went and usually end up talking for quite a while. Many times this is done over the phone.

Q. *How easy is it to get teachers involved who don't volunteer?*

Mrs. M. There has to be some pressure from somewhere or you don't change. There are some teachers who are happy with what they are doing and without pressure from somewhere, they are unlikely to change.

Q. *What is the impact of traditional constraints on the curriculum such as ISTEP (the Indiana Statewide Basic Skills Test) and the textbook?*

Mrs. C. We talk about such constraints in workshops, discussing how teaching problem solving supplements the text and helps prepare students for ISTEP.

Q. *Do you have any final comments?*

Mrs. M. Although there are committed teachers at all levels in the school district, I have generally found it easier to work with new teachers rather than those who have been teaching math for 20 years.

Mrs. C. I like the problem solving resource books that we developed along with project staff. However, I would like to see one more book added on using manipulatives to teach problem solving.

Table of Means and Standard Deviations for Each Administration of the IMBS for Regular and Special Education Teachers at Highland Park.

	August 1988 (*N*=19)		August 1989 (*N*=17)		August 1990 (*N*=14)	
Scale	Mean	(SD)	Mean	(SD)	Mean	(SD)
EFFORT	19.84	(4.72)	19.24	(4.07)	20.38	(4.84)
WORD PROBLEMS	21.68	(2.87)	22.12	(3.04)	23.07	(2.84)
STEPS	18.53	(3.32)	20.47	(3.89)	21.85	(2.91)
UNDERSTANDING	24.05	(2.30)	25.00	(2.65)	25.57	(2.10)
DIFFICULT PROBLEMS	20.26	(4.41)	19.94	(3.49)	23.00	(2.54)
USEFULNESS	25.47	(2.06)	24.76	(2.73)	26.50	(2.68)

Appendix C: Five Highland Park Teachers

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Overview

The focus of the school component of the PETTM project was to work with teachers at Highland Park Elementary School in Bloomington, a demonstration site for products developed by the project. . (For the first two years of the project, the Highland Park students and teachers were housed in a different building known as Dyer Elementary.) Highland Park was also the site of most early field experiences for preservice teachers (PSTs) enrolled at Indiana University at the time the project was taking place. As documented in other parts of this report, the project purchased manipulative materials for Highland Park school and paid for a mathematics resource teacher to work with Highland Park faculty members. One full day before school started in each of the three years of the project was devoted to inservice for the Highland Park faculty. During the first year, this inservice was planned by PETTM staff. By the third year, Highland Park staff were also heavily involved in the planning of the inservice day. Additional group inservice was provided through two sets of workshops for area teachers, including those from Highland Park . These workshops were funded by grants from the Indiana Department of Education and from the Indiana Commission for Higher Education. Additional detail on inservice activities can be found in the School Component chapter of this report.

To document the effectiveness of the inservice education for Highland Park faculty, several evaluation procedures were used. Note that inservice is construed broadly here, including workshops, informal and formal group meetings, and individual "coaching" from the resource teacher, Elsewhere in this report, students' performance on standardized tests and special problem solving tests is documented. Teachers' reactions to the project were continually solicited and have also been included in this final report. Tests and questionnaires, however, can only paint part of the picture of the effectiveness of an inservice project. **This portion of the report describes a qualitative study to document the teaching effectiveness of five teachers who were involved in the project.**

The five teachers described here were observed while they were teaching. In addition, they were interviewed about how the PETTM project had affected them. Observations and interviews were conducted by a member of the project staff. Guidelines for the observations included explaining the current instructional style of each teacher and documenting, through the interviews, the influence of the project on each teacher's style. To the extent possible, no judgments were to be made about the effectiveness of the five teachers. All teachers were shown the descriptions written about them and allowed to make changes. They were also given the option of having the descriptions deleted from the report although none chose to do so.

Observations of one teacher took place late in the second year of the project. The other four were observed during the third (final) year. The teachers chosen were all ones who were involved in the project and thus were not representative of the Highland Park faculty in general. In fact,

four of the five had been involved with project staff in the writing of resource books during the summer. When selecting individuals to be observed, it was felt that there was little value in observing teachers who had been only tangentially involved in the project and thus these five were selected from the group that actively participated in project activities.

This report is organized into five sections, one for each teacher. Although efforts were made to have uniformity in the observation procedure, the teaching styles of the individuals observed were so different that no single format was effective for documenting the influence of the project on them. Thus, the information provided on each individual is not always parallel.

Mrs. C: A First Grade Teacher

This report describes observations and discussions with Mrs. C, a first-grade teacher at Highland Park Elementary in southern Indiana. The focus of the study was the teaching practices of Mrs. C and how they relate to the influences of the Preparing Elementary Teachers to Teach Mathematics Project (PETTM), in which she was involved. The information was collected between October 23 and November 20 of 1990 and consisted of five classroom observations, two semi-structured interviews, and some informal conversations with Mrs. C.

In the first part of this report, I present a view of Mrs. C's classroom environment, focusing on mathematically related aspects. I look at physical aspects, internal dynamics, and the mathematical activities of the class. As will be shown, Mrs. C's classroom environment offered the students both visual and tactile stimuli. Also, the dynamics of the classroom provided students and teacher with a broad range of meaningful educational experiences. It is my view that a full understanding of Mrs. C's classroom environment can be reached only by viewing the physical aspects, the internal dynamics, and the mathematical activities of the class as an integrated whole. However, for the sake of clarity, these aspects are presented in three separate sections.

The second part of the report contains Mrs. C's reflections on her teaching practices and the changes that have happened because of her involvement with PETTM.

The Classroom Environment

Physical Aspects

Mrs. C's mathematics classroom environment offered a variety of visual and tactile stimuli. Hanging on the wall were several cards illustrating the ideas of quantity, counting, and time measurement. These ideas came up in the daily activities of the class. A computer was available although it was used mainly for drill and practice on addition and subtraction. Computer activities related to mathematical concepts were developed in the "math lab." In those activities, addition and subtraction were studied through the concepts of more than, less than, and equal to.

Mrs. C's students made consistent use of concrete materials in problem-solving and skill-related activities. During one observation, the students were using scales to weigh sets of unifix cubes. Sometimes Mrs. C substituted a concrete representation of a certain object or attribute with another kind of concrete representation. For example, on one occasion Mrs. C used pumpkins to represent popcorn balls in a problem-solving situation. (This representation will be explained in the problem-solving section of this paper.) It seemed to me that the students felt comfortable with the notion that objects can be represented by other objects or symbols, an important step toward their later understanding and appreciation of context independence in mathematics.

Internal dynamics

There was very little boredom in Mrs. C's classroom. The students experienced school through a multitude of activities. Many of these were small group activities taking place simultaneously in the classroom. Sometimes, I would get as involved in watching a group of students as they were involved in working. If it hadn't been for my duties regarding this study, Mrs. C would probably have been ignored!

Mrs. C's classroom dynamics offered a wide variety of settings. The students would at times sit on the floor close to Mrs. C and listen to her tell a story. This would then be followed by their writing about their favorite part in that story. At other times Mrs. C taught reading to a small groups of students. Meanwhile, the rest of the class would be involved with many different activities: pairs of students might read stories to each other, a group of students might be involved in a mathematical activity such as weighing pumpkins, an individual student might be playing with the computer, and other students might be working on the mathematics problem of the day, a non-routine process problem.

Sometimes, Mrs. C walked around the students' desks, observing and asking questions. Generally, when students asked questions, she asked them to think for themselves to answer those questions rather than to rely on her as the authority. At times, other students were asked to provide ideas and suggestions. In all cases, Mrs. C taught from a "problem-solving" orientation in that she expected students to use discussion and experimentation to construct their own answers to most questions.

Throughout my observations of Mrs. C's classroom, I noted an atmosphere of mutual respect, responsibility, and sense of humor. When cooperation from the students was needed, it was asked for in a very delicate way. When that cooperation was obtained, Mrs. C told the class how she appreciated their attitude. It is possible that early exposure of the students to such an open educational experience had provided for their mature patterns of intergroup negotiation.

Mathematical Activities

Three distinct types of activities were observed in Mrs. C's classroom. The first one, the weighing activity, involved individuals or pairs of students. It was generally developed during the period when the students were dispersed, working on several activities. The second activity were related to the development of subtraction skills and the third, a problem-solving lesson, dealt with problems having more than one solution.

Following are some excerpts of each of those activities.

Excerpt One (Weighing Pumpkins). M was weighing a pumpkin using a two-pan scale and unifix cubes. First, he balanced the pumpkin with the cubes; only then did he count the number of cubes.

At first some cubes were stuck together as in the figure below:

M counted them as one cube. Then he laughed and counted them as four cubes. Thereafter, he counted by disassembling the cubes that were stuck together. This process continued until he got 72 cubes for that pumpkin.

M worked on a second pumpkin. In contrast to the first pumpkin, this time he counted cubes as he placed them on the scale. M counted up to 64 cubes for the second pumpkin. Class continued with Mrs. C asking questions such as "Is it possible to find a pumpkin that weighs less than the one weighing 64 cubes."

Weighing pumpkins was an unstructured activity. It did not involve any formal data collection such as completing tables or worksheets. It was simply a time for the students to experience weighing. They could tell which, of two pumpkins, was the heaviest. They could balance, compare weights, and count the number of cubes.

The weighing activity for some students involved both counting forward and backward. One team of students counted the cubes as they were put on the scale. Another put too many cubes on the scale (e.g., 100 cubes) and counted backward as they removed cubes to make the scale balance.

Excerpt Two (Developing Subtraction Concepts)

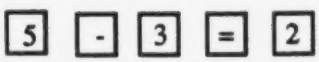
Two skill development activities, both involving subtraction, were observed in Mrs. C's classroom. In the first activity each child had a sheet of paper with pictures of bears and cubs. These animals needed some balloons. The children were asked to draw the balloons according to certain stories.

In one such story, Mrs. C told the class that the two cubs were each holding a balloon and that three balloons were flying away. The students were to tell how many balloons were in the picture. After the students answered "five," they were told to represent this story with a mathematical sentence. After the balloons were drawn, the students wrote the numbers and operations inside a row of squares as in this figure:

In another story, there were three balloons for the bears to hold and three in the air, totalling six balloons. The question referred to the number of balloons that flew away. The students drew the balloons and filled in the squares:

Having worked on those two examples, the students were asked to give a number bigger than two and smaller than nine for the quantity of balloons to be drawn. Next, they had to decide how many balloons would fly away and how many balloons the cubs would hold. Finally, they had to write a mathematical sentence for their story. The class worked diligently. Some asked "How many balloons?" to which Mrs. C replied, "That's your problem; the choice is yours."

I observed three students solving the problem in the following way. They chose 8 balloons, gave three to one cub and two to the other and had three fly away. However, in their answer, they subtracted the number of balloons that flew away from the number of balloons that the cubs had:



Mrs. C saw what the students did and repeated to the class that they have to count the total number of balloons in the picture and then subtract. "How many balloons did you start with? How many flew away?", she asked.

In the second skill development activity the students were given the following problem: A and S have five apples. If one of those apples is S's, then how many belong to A?

The students drew five apples.

Mrs. C read the problem again and one student answered: "four."

Mrs. C: Can you make a sentence for this problem?
Students: (writing: 5 - 1 = 4) Yes.

Another problem followed: M and S saw four pumpkins. None of them got smashed. How many are OK?

The students wrote down: 4 - 0 = 4.

The previous problem was changed to: If they (M and S) would divide them (the pumpkins) equally, how many (would each of them get)? This time the students didn't write a mathematical sentence. They just wrote the answer 2.

During these exercises, I noticed that pictures of the situations were drawn first. Then, a letter was assigned for each person in the picture. Next, the subtraction was carried out. And

finally, the sentence was written down. The activity developed naturally, in a very informal and relaxed way.

Excerpt Three (Problem Solving)

These activities were associated with the problem of the day. This problem came from the *Problem of the Day for Primary People* book developed by Mrs. C and others as part of PETTM. Students worked on the problems at their own pace, individually or in groups. During the class discussion, they presented their solutions and compared them with the solutions generated by other students. Following are some examples of that work:

The first problem to be discussed was the following: In how many different ways can you spend 7¢ to buy pumpkins, cats, ghosts, and witches?

(picture of ghost)	(picture of pumpkin)	(picture of cat)	(picture of witch)
2¢	3¢	4¢	1¢

Mrs. C wrote on the blackboard "There is more than one answer. Try to find all of them. You have 7¢ to spend. What will you buy?" The discussions regarding five distinct solutions for this problem went as follows:

Solution #1

M:	I can buy the four of them.
Mrs. C:	You can buy [them all] ... right?
L:	You can't get every four of them.
Mrs. C:	Why? Why is 7¢ important?
A:	Is money!
C:	It's a lot of money!
T:	Is all you got.
Mrs. C:	You have to spend **all** your money. You can't have any left.
M:	Cat and pumpkin.

Mrs. C wrote M's solution on the blackboard:

(picture of cat)		(picture of pumpkin)		
4	+	3	=	7

Solution #2

K: Pumpkin plus witch.

Mrs. C wrote K's solution on the blackboard:

(picture of pumpkin)		(picture of witch)		
3	+	1	=	4

Mrs. C: How much money is left.
Students: 3¢.
Mrs. C: Can we buy something else?
V: Pumpkin.

V's solution was written:

(picture of pumpkin)		(picture of witch)		(picture of pumpkin)		
3	+	1	+	3	=	7

Solution #3

Mrs. C: (referring to D) D wants to buy a pumpkin and a ghost. D wants to buy another ghost.

D's solution was written:

(picture of pumpkin)		(picture of ghost)		(picture of ghost)		
3	+	2	+	2	=	7

Then, Mrs. C used unifix cubes as prices. Each cube was worth 1¢ so seven cubes were used to do the problem.

D: Is that it?
Mrs. C: I don't know.

Solution #4

Mrs. C: What else could I buy?
D: A ghost, a pumpkin, and a witch.

Mrs. C wrote D's solution on the board:

(picture of ghost)		(picture of pumpkin)		(picture of witch)		
2	+	3	+	1	=	6

Mrs. C: (counting) Six. What else can I buy? I have 1¢ left.
Students: A witch.

Mrs. C then completed that solution.

Solution #5
R: One ghost and one cat.

R's solution was written on the board:

(picture of cat)		(picture of ghost)		
4	+	2	=	6

Mrs. C: How can I make this 7?
Students: A witch.

While completing R's solution, Mrs. C told the students that "there are more ways of doing this problem but we are not going to do them today."

Mrs. C included the names of her students in problems of the day. In a conversation, she told me that she felt strongly that using students' names in problems was a good teaching technique. Mrs. C's students enjoyed being in problems and sometimes acted out their part during the discussion. The following problem provides another example of Mrs. C's teaching style: A and B have 6 popcorn balls. A has 2. How many does B have?

Mrs. C: What do you know?
L: A has two.
Mrs. C: Two what?
L: Balls of popcorn.

K thought that B had six balls of popcorn and A had two. T thought that A and B had six balls of popcorn.

Mrs. C: Let's pretend that balls of popcorn (using pumpkins to represent them) are here. I'm not sure I understand what you mean.

One student was in charge of getting the pumpkins.

S: Two and five.

Mrs. C waited for somebody to react to S's solution.

K: If you add them up it will go to seven.
T: Two and six.
Mrs. C: Does the problem say that we have eight? What should we do?
Students: Take two away.
Mrs. C: How many now?
Students: Six.

A took two for himself.

Mrs. C: Now, how many does B have?
Students: Four.

Mrs. C read the problem again and the whole class answered aloud, "four."

A third example of how Mrs. C uses problems of the day can be seen in the following discussion of the problem: If there are five blue pencils and five red pencils what combinations of pencils could you make if you could only have five?

Before the class started discussing this problem, I saw some of their solutions. One student wrote the initials for the names of the colors, giving one combination of pencils. Another student colored five pencils using blue and red and provided one combination of pencils. M provided three different groups of pencils using abbreviations for blue and red:

b r b r b r r r b b b b r r r

The last two solutions are the same. It is possible that M considered the order of the colors as being an essential aspect of the problem. However, looking at his first two combinations, this cannot be taken for granted.

The students showed their solutions. R's solution (using unifix cubes to represent the pencils) was:

r r r b b

Mrs. C asked the students to write down R's solution. She made a table on the blackboard with headings "red" and "blue."

C's solution (using the unifix cubes): r r b b b.

Mrs. C asked T to offer a solution which was different from the ones on the blackboard.

T's solution with the unifix cubes: r r b b b.

Mrs. C: We already have this.

T's new solution: b r r r r.

The next student N, gave the solution: r b r b r. This solution was only a different permutation of a combination already given. N changed the arrangement of the colors (using the unifix cubes). The combination remained the same:

r b b r r

N tried again: b r b r b.

Mrs. C: How many?
S: Three blues and two reds.
Mrs. C: We need a new picture (combination).

W showed two new solutions: b b b b b and r r r r r.

Mrs. C: Now, there is one more.

While trying to find the missing solution, some students picked less (or more) than five pencils.

Mrs. C: Look at the numbers (in the table). Is there a pattern there? Look.

W provided the missing solution: r b b b b.

The complete table written on the blackboard was:

red	blue	
3	2	R
2	3	C
4	1	T
0	5	W
5	0	W
1	4	W

Mrs. C called the students' attention to the fact that the order in each pair of numbers can be reversed. The table was reworked to show one column increasing from 0 to 5 while the other was decreasing from 5 to 0. The whole class worked on this activity, which for some students was not clear yet. The new table looked like this:

red	blue
5	0
4	1
3	2
2	3
1	4
0	5

Then, a student said with wonder: "Oh, you got 0, 1, 2, 3, 4, 5." Then Mrs. C commented on the number of different pairs they got (six).

Mrs. C's Reflections About the PETTM Project

Mrs. C. told me that prior to her experience with PETTM, she wasn't happy with the way she taught. She did not know a lot about problem-solving strategies and did not feel comfortable

using those strategies in the classroom. She needed more exposure and experience in teaching problem solving.

Mrs. C mentioned that the workshops were both excellent and relevant, and that discussions on mathematics were lacking before the workshops. It was through the workshops and in conversations with her colleagues that she started hearing different kinds of comments and questions, thus getting challenged to deepen self-reflections on teaching practices. Now, Mrs. C feels involved in the sharing of innovative practices among the group of teachers with whom she is working.

Mrs. C told me that Jackie Gorman (the mathematics resource teacher provided by PETTM) was very helpful in the development of the Highland Park Elementary school teachers involved in the project. This point was stressed by Mrs. C when she noted that teachers' access to support people to whom they can address questions is important. As she expressed it, "That's (personnel and material resources) what made it for us."

According to Mrs. C, the teachers who got involved in the project should communicate their experience to other teachers. They should support the teachers who participated in the project and others interested in getting that experience. Currently, Mrs. C and some other teachers are meeting every two months with teachers at Highland Park and at other schools in the district. The purpose of the meetings is to share experiences. Mrs. C observed that there was still a lack of communication among the teachers.

Regarding the lack of communication problem, Mrs. C thought that a better strategy would be to have the Highland Park teachers and the I.U. personnel work together. In her view, the I.U. personnel have to be sent to a school to see what happens, what works, and what doesn't work. They need to go back to the teachers they observe and follow up on their classroom activities to get a true feel for classroom practice.

Reflecting on the district adopted textbook (*Mathematics Today, Level 1*, published by Harcourt Brace Jovanovich), Mrs. C told me, "Before the project, I would follow the book from the beginning till the end. Today, I prepare the kids to take the book test and that's all."

Mrs. C does not require students to do every page dedicated to the practice of skills. Instead, students have a math bag from which they pick a practice page once a week. Skill-related activities that are not introductory are included in the math bag. The students who are slower in performing routine computations are pushed a little harder by Mrs. C in an effort to "keep them going." Most react very well. Mrs. C commented that people mature at different rates and that there is no need to make students anxious about mathematics.

Mrs. C's planning is based on the Monroe County Community School System (MCCSC) curriculum. She plans her classes by first defining what she needs to teach and then deciding where she can find the material or ideas to work on those needs. This approach illustrates the

influence the project has had on Mrs. C's planning. Initial ideas for "non-text" teaching methods were generated in the PETTM sponsored workshops. The ensuing discussions with colleagues and project personnel further supported the implementation of those ideas.

Generally, Mrs. C has her students work on exploratory activities (e.g. weighing pumpkins) and the problem of the day every morning. Skill-related activities (e.g. introductory lessons and math bag sheets) are developed in the afternoons. Mornings are reserved for informal exploratory activities; afternoons for the formal work. Usually, whole-class discussions are held in the afternoon.

The activity described previously that involved bears and balloons is an example of an introductory lesson. Referring to it, Mrs. C told me that she still spent some time doing "boring stuff," which became less boring when done with manipulatives: "It is amazing how the children get to understand it with the materials." Currently, Mrs. C is involved in plans for merging introductory lessons and problem-solving activities.

Mrs. C was one of two principal authors of the resource book *A Problem A Day For Primary People: A resource book of daily process problems for students in grades 1-3*. According to her, that enterprise was frustrating at times but in the end "we had a marvelous return." She is concerned with some needed changes in the book and is sure that soon it will be revised.

Work on the book forced the authors to find problems with different strategies and with too much or too little information. Concurrently, the authors had to take into account the MCCSC curriculum. Mrs. C noted that these two points of view were at times incompatible as the MCCSC curriculum was based on skills more than on thinking and problem solving.

To Mrs. C, mathematics is a sharing experience in which students help each other. In my view, this happens with every activity in Mrs. C's classroom. Her contact with the project has changed her and her students' perspectives about mathematics. Prior to PETTM, Mrs. C's students saw mathematics as a mandatory subject which required substantial memorization and creation of a single correct solution to fairly routine problems. Currently, Mrs. C's class is working on some strategies to solve problems. And, as presented earlier in this report, students are getting a feel for the existence of different solutions in some problems they solved. Also, problems with too much or too little information are being discussed. According to Mrs. C, "You have to stretch them to look for more than one way of doing a problem. Now, we are working on problems that have too much or too little information. The kids still tend to use all the numbers that are present in a problem."

One of the strongest influences the project has had on Mrs. C's class is to have the students "verbalize" about mathematics. The verbalization allows Mrs. C to better understand what each student is thinking. This, in turn, allows her to guide her questions and comments to improve student comprehension rather than simply memorize mathematics. Mrs. C also notes that

verbalization among students helps them to understand new concepts.

Conclusion

The interviews I conducted with Mrs. C represent her viewpoints regarding the impact of PETTM on her teaching practices. The observations show a teacher whose classroom environment is in agreement with her stated perspectives on good teaching.

Mrs. C is experimenting with the implementation of new ideas in her classroom. The workshops provoked her to reflect about her practices as a teacher; the sharing environment provided by PETTM allowed her to search for mathematical approaches that she thinks are more meaningful for her students.

Currently, Mrs. C approaches mathematics teaching by searching for an optimal combination of **experimental**, **problem-solving**, and **skill-related activities**. Although she considers the MCCSC curriculum in her planning; she feels free to implement ideas and materials that give that curriculum a distinctive characterization. The textbook, although not completely abandoned, was often avoided in favor of the math bag, the problem of the day book, and other exploratory activities.

Prior to PETTM, Mrs. C's students learned mathematics by memorizing it. That emphasis has been replaced by the encouragement of creativity. Mathematics as merely a mandatory activity was transformed into a living experience. Although Mrs. C's students often continue to look for the answer when solving problems, they have been exposed to an environment that encourages them to consider the differences that exist among themselves as people and, therefore, as creators of distinct solutions to those problems.

Mrs. K: A Second Grade Teacher

The data for the study of Mrs. K, a second-grade teacher, consisted of two classroom visits and several discussions during the spring of 1991. More visits had been planned but circumstances beyond the control of either the teacher or the observer resulted in their cancellation. There were 18 students in Mrs. K's class. Although instruction in several subjects was observed, the study was focused on mathematics and thus most observations took place during mathematics periods.

The body of the report is divided into four sections. The first one consists of a description of the classroom environment. Mrs. K's classroom activities are the subject of focus in the second section. In the third section I focus on affect and patterns of behavior that could be observed in that classroom. The final section involves a summary of the conversations I had with Mrs. K.

Classroom Environment

The display on one of Mrs. K's classroom bulletin boards, called "Counting Money," shows the five common United States coins and their associated monetary values. Also, equivalences such as 5 pennies = 1 nickel and 4 quarters = 1 dollar are represented. There is a number chart with the first hundred numbers (in a 10 by 10 grid) hanging on the wall near the blackboard. Jackie Gorman (the mathematics resource teacher) used this chart once when she went to the classroom to "play" games with the children that involved even and odd numbers and other number patterns. A big clock, drawn on a piece of paper, is also hanging on one wall.

Several of Mrs. K's students were considered "at risk." Some of these individuals were still adjusting to the social norms of school and thus would have been difficult for any teacher to work with. These students often left class to work with social workers and other special personnel in the school. In discussions with Mrs. K, she said that her children ranged from average to low average in intelligence. She also said that the children's behavior was a major influence on her teaching methods. The school's mathematics specialist (Jackie) had worked with Mrs. K's class and confirmed that there were a large number of "behavior problems" in the class.

Classroom Activities

Mrs. K focused primarily on review activities during the period of my observations in her classroom. Those activities were associated with addition and are exemplified in the following two excerpts.

First Excerpt: During one visit to Mrs. K's classroom, I observed a lesson in which she reviewed addition with money. Three activities were developed involving the children's knowledge of

money and addition. Initially, Mrs. K showed the children five large paper coins. The children had to identify each of the coins along with the monetary value and president pictured on each coin. After some moments of guessing and suggesting the children provided the correct associations:

penny (Lincoln)	-	1¢
nickel (Jefferson)	-	5¢
dime (Roosevelt)	-	10¢
quarter (Washington)	-	25¢
half-dollar (Kennedy)	-	50¢

The children's eagerness to guess answers was apparent. I noticed that some children were disappointed and complained to Mrs. K when they were not chosen to answer a question.

The next activity of the day consisted of Mrs. K picking up some large paper coins, showing these coins to the children, and asking them how much money she had. Some examples follow:

Example 1. Mrs. K showed four paper coins: a half-dollar, a dime, a nickel, and a penny. One student answered: " 66." When asked to explain his answer, the student counted aloud pointing to the coins: "fifty ... sixty ... sixty-five ... and sixty-six."

After the first example Mrs. K selected one child at a time to come to the front of the room and play the role of the teacher for the rest of the class. The student chosen would choose the coins to be used and select someone to give the answer. The students were quite eager to participate in this type of activity.

Example 2. A student answered "a dollar" when one half-dollar and two quarters were shown.

Example 3. A different counting procedure was used by one child, J, when he was shown a half-dollar, two quarters, and a dime. First, J added the values of the half-dollar and the dime. Next, he added the value of the two quarters (25 + 25 = 50). J's final result was obtained by adding the two partial sums: "Fifty ... sixty ... and this is fifty [referring to the value of the two quarters]. A dollar." Although J's procedure was elaborate he made a mistake in the final addition. J's mistake was promptly corrected by his colleagues who shouted "a dollar and ten."

Following this whole class activity, the children were given a sheet of paper having some groups of coins drawn on it. They were again supposed to identify the total value of the coins. Some children worked alone and others worked with others around them. Mrs. K appeared to

approve of children working in groups although she did not tell them to do so. A group of girls asked me what they were supposed to do in the paper and pencil activity. I asked them if they could read the problems. One girl replied with a yes. Then, she read the first problem, remained silent for a moment, and gave the answer to that problem. When I asked her what she did, she told me that she had counted the coins. After that conversation, her group got together to work on the other problems.

Second Excerpt: During another observation, Mrs. K's class was reviewing addition with regrouping. A few addition problems had been written on the blackboard. Mrs. K asked some students to work on them, telling the students that one of the problems did not need regrouping. She also reminded the students of the place values of the different digits.

J worked on the first addition:

$$\begin{array}{r} 83 \\ +5 \\ \hline 88 \end{array}$$

F went to work on the next addition:

$$\begin{array}{r} 39 \\ +43 \\ \hline 2 \end{array}$$

Mrs. K: What is 9 and 3, F? You put the 1 in the tens column.
After some seconds, F wrote down 82.

H did his very fast:

$$\begin{array}{r} 75 \\ +5 \\ \hline 80 \end{array}$$

Next came M:

$$\begin{array}{r} 15 \\ +26 \\ \hline 31 \end{array}$$

Mrs. K started checking M's answers. At this very moment, M interrupted her and wrote a 4 in the place of the 3, giving the answer 41.

Mrs. K told the students that she was very pleased with the way they worked on these additions. As a prize for their success, she announced to them that they were ready to work on

page 176 of the textbook (*HBJ Mathematics Today*, 2nd grade). In that assignment, the students had to do 45 addition problems, only some of which required regrouping. Most of the students, did not seem interested in the assignment. Some children started to play around and others worked on assignments from other subjects. Mrs. K told me later that the students had seat work in several subjects and that sometimes they could choose the order in which they completed the assignments. Mrs. K stopped by the desks of students who were not working on mathematics and opened their textbooks, asking them to do their math first. Then, she went to check whether the directions in the textbook were understood by the individual students.

While the students were working, I sat near a student and observed what she did. M, a special education student who spent part of her day in Mrs. M's class, was working on the addition exercises. At first, there seemed to be a pattern in her answers:

22	41	23	45
+56	+27	+16	+4
88	78	49	59

Her next answers, however, showed that her misunderstanding of addition went beyond that pattern:

33	29
+44	+61
88	91

While M worked on the last two examples, I could observe that she wrote the tens digit before she wrote the units digit. I was still confused about M's addition algorithm so I continued observing her work:

35	75
+50	+5
95	81

After these examples, I kept asking myself about M's patterns. When would she add one to both columns? Did she consider the two columns as independent from each other?

When M finished the assignment, she went to show it to Mrs. K who examined the paper and noted the abundance of errors. Mrs. K knew that M was having difficulties and that overcoming those difficulties would take a substantial amount of time. Rather than take too much time from the rest of the students in the class, Mrs. K told M to work with the tutor in the library.

Dealing With Behavior Problems

During my observations, I noticed that, with a few exceptions, the children seemed relatively calm. Their general behavior was not much different from that of the other classes I had observed. Mrs. K's rule was that children were given two warnings for unacceptable behavior. If they did not comply, they were sent to the office to do their work alone.

Mrs. K allowed the children to talk softly with their colleagues. Occasionally, some of them expressed their feelings by singing and walking around. Although the children seemed very energetic, most of them were able to concentrate on their assignments.

Mrs. K was affectionate with her students. On many occasions her students would seek her for closer contact or just to get her attention. They were encouraged by her tenderness and words of praise. There were times, however, when Mrs. K got upset with some of the students. In those cases, she would have them seated apart from their colleagues so that they could reflect upon their behavior. I witnessed two particular occasions where Mrs. K openly dealt with inappropriate behavior.

The first occasion was one in which a group of girls was working on word search puzzles. The girls were excited about the game and the discussion got too loud to be appropriate. Mrs. K told T that she would have to stay in the room for ten minutes after the class was over. T expressed displeasure with such a reprimand, telling Mrs. K that she was looking for the missing word.

The second occasion involved a single student, J. J had called a colleague a bad name. Mrs. K reacted by telling J that he had to stay in the classroom for ten minutes after the class was over. This way he could think about his behavior. J seemed angry and disappointed: "I hate this class. I don't care." He walked around for a few moments and sat on his desk, without even looking at a mathematics assignment he was supposed to do.

After having observed that J was still not working on the assignment, Mrs. K asked him to leave the room. J did not want to leave and agreed to begin working on the mathematics assignment:

J: Okay, I do it.

Mrs. K: But this is the last time. You are not following directions, J.

J, in reality, did not change his attitude toward the assignment. He remained quiet but worked on something else.

Ten minutes later, J showed Mrs. K what he had been doing. Mrs. K examined J's paper and praised him. Next, she said: "I bet you can do more than this. I bet you can." J was pleased.

However, Mrs. K's words of praise did not work as she had expected. Some minutes later, Mrs. K sent J to the school office as he remained unwilling to work on the mathematics

assignment. Reluctantly, he left the classroom. I found out later from Mrs. K that J had been in a special education class since kindergarten and was still adjusting to behavior expectations in her classroom.

As J's episode developed, the other students were busily involved in that mathematics assignment. Mrs. K walked around the students, engaging in individual conversation with each student. That was the general pattern of communication in her classroom.

Conversations with Mrs. K

I spoke with Mrs. K on three occasions over the course of the month in which observations took place. We discussed a number of topics, the most important of which will be reported here.

Student Ability and Problem Solving. During my interviews with Mrs. K, she noted that she perceived her students as intellectually slow and low in motivation. Such a perception pervaded her behavior as a teacher. She felt that the best way to get the students' intellectual attention and, simultaneously, respond to their needs for affection was to work individually with them if they needed more explanation. She also expressed her perception when discussing classroom problem-solving experiences by saying "You have to teach them how to think."

In another discussion of problem solving, Mrs. K noted that she was not spending time on extra problem-solving activities. She commented that such activities had been developed with her students in the previous year. Her current class, however, included many students with serious reading deficiencies, little parental support, and low motivation. All of these factors contributed to her decision to deemphasize problem-solving activities. Mrs. K noted that she would be happy if her students would just understand "the very basic stuff."

Mrs. K's reluctance to include problem-solving activities in the classroom was also observed in her dependence on the traditional second grade textbook. Classroom and homework assignments were taken from that textbook. At one point, Mrs. K mentioned the teachers' resource book developed by the project, *A Problem A Day For Primary People: A Resource Book of Daily Process Problems for Students in Grades 1-3*. She told me that she liked the newly revised edition and favored its adoption. However, she felt the book was most valuable for higher achieving students. Mrs. K also mentioned that she was interested in seeing how the "commercial" textbooks would change their current approaches to include a greater emphasis on problem-solving.

Grouping of Students. At one point, Mrs. K and I discussed grouping students. Although there were instances such as those previously mentioned, in which students did work together, Mrs. K said that individual work was the general approach utilized during classroom assignments. Group

work was not encouraged because "they [the students] don't do what they are supposed to, even when they work in pairs."

Use of Manipulatives. During my observations, Mrs. K did not have her students work with concrete materials other than paper coins. However, she told me that her approach toward teaching new concepts had changed as a result of the influence of PETTM. Now, she would introduce new concepts, such as subtraction, by demonstrating them with concrete materials, such as base ten blocks. Those demonstrations would allow her to provide her students with visual representations of the procedures that were explained on the blackboard.

Evaluating Student Learning. The evaluation of Mrs. K's students is mostly based on classroom assignments and tests taken from the adopted textbook. Referring to the textbook, Mrs. K told me that the students do not have to work on all the problems in a section if they understand what they are doing. As she explained to me, "I can't see why they [the students] have to do fifty problems when they show that they can do ten!" Mrs. K also noted that a number of students did not return homework. She felt that parents were at least partly to blame for this as they did not actively encourage students to complete assignments at home.

Every day Mrs. K evaluates what her students do in their classroom activities. She explained to me that by daily individual contact with the students, she has become aware of the students needing extra help. When the students failed a test, she went over the problems they missed, individually, to reteach and show them their errors. She did express, however, feelings of uncertainty regarding this process. "Hopefully, when they [the students] see it [the concept] in the following year, they will master it."

Involvement with PETTM. My conversations with Mrs. K revealed a teacher who had been involved in the activities of PETTM, although not as heavily involved as some other teachers. She told me that she went to the workshops but could not find the time to reflect and elaborate on the materials which she obtained during those workshops. She also complained about the lack of time that teachers had to meet with each other for the purpose of discussing those materials. As she put it: "The project has probably changed some of my attitudes and methods. But I'm struggling to apply them. It's frustrating." Mrs. K guessed, however, that the teachers who got more involved with PETTM were already using new methods and applying new ideas in the classroom.

Mrs. K asked me about the continuation of the project and the integration between the I.U. personnel and the school teachers. She told me how frustrated she felt with the end of the project getting closer. "That's the way it always is. Teachers are often exposed to new concepts, but usually followup tends to be weak." On the other hand, she believed that not everything was lost:

"Some teachers picked up parts of it [the project]." She further indicated hope that the textbook would change to reflect new attitudes toward problem solving and its importance.

Conclusion

Overall, my observations and conversations show that PETTM has had an impact on Mrs. K's teaching practices, but the impact has been less than it has been on some other teachers. Mrs. K demonstrates mathematical concepts with manipulatives and she is aware of resources for teaching non-routine problem solving. She is "in touch" with her students in the sense that she knows that simply explaining a concept is not enough for them to master it. Several factors could account for why Mrs. K was less involved in PETTM than some other teachers.

The first potential reason for the limited impact of the project on Mrs K's teaching practices is the lack of time to reflect and elaborate on the workshops that she went to and the absence of meetings with teachers in which the topics related to PETTM could be discussed. From Mrs. K's point of view, the teachers who were influenced the most by PETTM were the ones who got more involved with it through additional summer and school year activities.

Mrs. K's strong concern with the overall behavior of her students may also be considered as a potential reason for the limited exercise of the PETTM philosophy in her classroom. While discipline was not intentionally ignored by project personnel, it was not a major focus of the project. It was assumed that discipline could be maintained while problem solving instruction was taking place. Mrs. K had a very difficult group of children to work with and thus failure to place more emphasis on behavior management might be considered as a weakness of the PETTM project.

Another reason for the limited impact of PETTM on Mrs. K's teaching practices lies in the serious reading deficiencies, lack of motivation, and the general lack of parental support for Mrs. K's students. Discussions on how to work in a classroom with these limitations were not addressed in project sponsored workshops as they were not PETTM's primary goal. At least in Mrs. K's case, more emphasis on how to teach problem solving to students with weak reading skills and from academically unsupportive homes may have been beneficial.

In summary, Mrs. K was touched by the ideas and approaches presented in the project sponsored workshops. Indeed, she is a dedicated professional who felt frustrated by not being able to use project ideas as completely as she wanted to. She has made changes in the way she teaches and is constantly looking for ways to reach more students. The PETTM project was a valuable experience for her but it failed to adequately address techniques for overcoming what she perceived to be major barriers to learning: low motivation, lack of discipline, and weak parental support.

Mrs. B: A Third Grade Teacher

This section of the report is based on a series of observations and interviews which focused on the teaching practices of Mrs. B, a third-grade teacher at Highland Park Elementary. The study was conducted between October 23 and December 3 of 1990, about six months prior to the conclusion of the project. There were six observation of Mrs. B's class. Four of those were followed by loosely-structured interviews, conducted in an informal and relaxed atmosphere. Additional information was collected during other brief conversations with Mrs. B.

The main part of this report is focused on Mrs. B's reflections regarding her teaching practices and the extent to which PETTM has influenced those practices. Examples are provided to represent Mrs. B's classroom methods. Many details of Mrs. B's teaching practices were given special attention throughout the study.

The report is divided into two sections. The first section consists of a description of the teaching methods as observed in Mrs. B's classroom. The second section is based on the interviews and contains Mrs. B's considerations about her teaching practices as they relate to the PETTM project.

Description of Mrs. B's Teaching Methods

Introducing Concepts

When introducing new mathematical concepts, Mrs. B often used traditional direct instruction methods such as giving directions, being the main questioner, and using a product-oriented approach. The questioning process focused on the students' explanations and served as a vehicle for getting students to think ahead to the next idea to be presented. The class was conducted with an emphasis on discipline. Students were not allowed to speak among themselves and worked on individual assignments most of the time.

The non-traditional aspects of Mrs. B's instruction were demonstrated in several ways. As will be shown, students had the opportunity to explore the classroom in order to experience the concepts they were learning. Also, when questioned by Mrs. B, each student had the opportunity to express him or herself in a very open way. Finally, Mrs. B let the students struggle for themselves when facing difficult situations; generally, no direct suggestions were given.

Excerpt One (Length Measurement #1). Below are examples of the methods employed in developing the topic of length measurement. They are excerpted from two classroom observations. The first took place when Mrs. B and her students were developing the topic of length measurement in inches, feet, and yards.

There were eighteen students, sitting in groups of four or five. The students were measuring their desks and the table in the front of the class while Mrs. B was commenting on how measuring errors might arise. For instance, while some students were using their fingers or a

small ruler to measure the table, Mrs. B suggested that such a procedure could lead to errors and that, in such cases, it would be better to employ a larger measurement device such as a yardstick.

Some of the measurements obtained by the students were written on the blackboard, so that the whole class could discuss them. The example below represents a situation where feet and inches were combined. The students added 4 inches to 13 inches, obtaining 17 inches. Having learned the correspondence between feet and inches, they took away 1foot (12 inches) from 17 inches and got 5 inches. They stated that their answer was 17 inches, or 1 foot, 5 inches.

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 13
+4
 17
-12
  5
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On another occasion, students combined feet and yards. When Mrs. B showed a yardstick and asked "how many feet in this one yard?" they answered "three."

In a third example, the students had to transform 40 inches into other units. One student added 12 inches to 12 inches, then another 12 inches to the 24 inches already counted. Then, he converted the 40 inches into 3 feet and 4 inches and gave the answer 1 yard 4 inches.

After class discussion of the measurement activities, the students worked on a series of exercises proposed in the district adopted text *Mathematics Today, Level 3* (published by Harcourt Brace Jovanovich, 1985). During that practice, I noticed that when a student was asked to measure something, the teacher would wait for the answer. If the answer provided was out of a certain range, the teacher would ask that student to think about it. In some cases, that student would repeat his or her measurement; in others, his or her answer would be modified without additional measurement.

While the students were working on problems in the text, Mrs. B would oversee their work. Every time she observed an error on a student's sheet, she would ask the students what happened, and wait for the answer from anyone.

Excerpt Two (Length Measurement #2). The following week, the class continued to work on length measurement. Mrs. B showed a three-colored yardstick to the class, each color measuring 1 foot. She asked the students about the yardstick:

Mrs. B: What can I say about this ruler?
Students: Three feet.

Mrs. B: What else can I say about this ruler?
Students: One yard.
Mrs. B: What else can I say about this ruler?
Students: Thirty-six inches.

Mrs. B then wrote 36 inches = 3 feet = 1 yard on the blackboard and stated "A foot is approximately the distance from the tip of your fingers to your elbow. And a yard is approximately the distance from the tip of your fingers to your nose when your arm is outstretched." She then asked the students about the most appropriate unit to use when measuring the distance between the school and their homes. Some students made use of "close to" when referring to the approximate distance. The same question was applied to other lengths, such as the length of their arms, noses, a couch or room at home, the height of a soup can, etc...

Mrs. B was quite serious about having the students answer for themselves when asked. She didn't allow other students to interrupt or take the place of a colleague. They could answer only after she felt that she had worked enough with a particular student. This did not necessarily mean that the student who she questioned comprehended all the material. However, it made it possible for the students to express their current thoughts regarding the topic under discussion, thus allowing Mrs. B to keep track of their understanding.

The class continued with Mrs. B's commenting on the relationship between the units of measurement used and the size of the error, if any: "You can measure my desk with feet instead of yards but as we saw last time the more measurements you make the more you are off." The lesson continued and the students seemed very excited.

Mrs. B: Bed?
Students: Yards.
Mrs. B: Mom's car?
Students: Yards.
Mrs. B: Height of your desk?
Students: Feet.
Mrs. B: (after measuring the height of a desk) It's about two feet.
Mrs. B: The distance between Boston, Massachusetts and Tampa Bay, Florida: yards or miles?
Students: Miles.
Mrs. B: The length of a swimming pool. Yards or miles?
Students: Yards.
Mrs. B: The length of a leaf. Inches or feet?

Students: Inches.

In another type of exercise the students compared different units.

Mrs. B: How many inches are in four feet?
E: Eight.
Mrs. B: If that were the case each foot would be two inches, right E? How many inches are in a foot?
E: Twelve.
Mrs. B: So, how many inches are in four feet?
E: Forty-eight.
Mrs. B: How do you know that?

E writes on the board:

12, 12 + 12 = 24, 24 + 24 = 48.

Mrs. B: With that in mind, how many inches are in seven feet?
A: Eighty-four.
Mrs. B: How did you get that?
A: 48 from the last exercise (writing on the board: 12, 24, 36). So,

$$\begin{array}{r} 48 \\ \underline{36} \\ 84 \end{array}$$

Mrs. B: (Asking A) What are other ways?
A: I put seven twelves down.
Mrs. B: O.K., or seven times twelve.

In an interview, Mrs. B told me that A had already had multiplication in the previous year. She called on him in order to see him employ it explicitly, thus giving the class an example of the next topic to be studied. Although A's answer contained an idea of multiplication, she later noted that his comments turned out to be inadequate for Mrs. B's purposes.

Problem-solving activities

The second defining characteristic of Mrs. B's teaching method was the use of what I called a problem-solving approach. It was employed in two distinct kinds of activities.

Students solving problems. In this kind of activity, students worked on the solution of problems taken from the resource book *Problem of the Day for Primary People* created primarily by Mrs. B and another teacher who had worked closely with PETTM staff. Although there were few mathematics related posters or pictures on Mrs. B's bulletin boards and classroom walls, every day one could find a "problem of the day" section on the blackboard. That section, consisting of mostly problems from the resource book, was incorporated into the daily set of activities for the students.

Mrs. B seemed very confident regarding her teaching with respect to problem solving. She believed there to be more than one solution and more than one method of solution for many problems. In addition, she realized that since people are different from one another their ways of solving problems might also be different.

As part of her problem-solving orientation, Mrs. B's encouraged students to discuss their mistakes and express their opinions about the process of solving a problem. This helped enable them to realize that there is much more to a problem than its final solution. As an example, Mrs. B talked about a day when her students worked on a problem in pairs. She observed that, while they were discussing the solutions, some of them mentioned that embedded within "wrong" answers were some correct aspects in the process of solution.

On another occasion, the students were given a problem that lacked information (problem 28 from *A Problem a Day for Primary People, Grade 3*). They told Mrs. B almost immediately that the problem could not be done since they didn't know what number to assign to the concept of "some." Later on, the students decided to use various numbers in the place of "some," finding answers that varied in accordance with those numbers. Mrs. B told me that her students were able to do that because they had done a lot of work in problem solving.

Two examples that illustrate the dynamics of problem solving in Mrs. B's classroom follow. In the first example, the students worked on the problem of the day, individually and in groups. Some of them wrote their ideas on the blackboard; others worked at their desks. When they finished, solutions were discussed as a group. Solution and discussion of a single problem took almost twenty-five minutes. The second example relates to an activity involving the use of concrete materials in the study of volume.

Excerpt Three (Problem of the Day). The problem of the day was as follows: Jennifer, Erica, Sarah, and Maria were stringing popcorn for their Christmas tree. While stringing the

popcorn, they were also busy eating it! Maria ate 15 more pieces than Sarah. Jennifer ate 13 more pieces than Erica. Erica ate 12 more pieces than Maria. Sarah, who was busy working, ate only 12 pieces. How many pieces did Jennifer eat?

Mrs. B: R, what did you get?
R: Twenty-five. Because Jennifer ate thirteen more pieces than Erica.
Mrs. B: How did you get twenty-five?
R: I added thirteen and twelve.
Mrs. B: How do you know that Erica ate twelve pieces?

A long silence followed.

C: He didn't look at more, (referring to the term 'more' that appears after 13 in the problem: Jennifer ate 13 'more' pieces than Erica).
Mrs. B: What is the difference? (talking to R) Do you see the difference between what you said and what C said?

R wasn't feeling comfortable with the possibility of having made a mistake.

Mrs. B: Don't get upset, R. It's O.K.

Mrs. B: (talking to the class) Do we know how many Erica ate? What **do** we know for a fact to start out? There are many facts up there.

The class was once again silent.

Mrs. B: What is one thing we know? We know, for example, that Jennifer ate thirteen more than Erica. What else do we know?
G: Maria ate fifteen more pieces than Sarah.
Mrs. B: What else do we know?
L: Erica ate twelve more than Maria.
Mrs. B: What else?
K: Sarah ate twelve pieces. [K solved the problem by working backwards. Mrs. B knew that but wanted the class to discuss the problem rather than have K show her solution.]
Mrs. B: What is it that K said different from the other facts?

C: We know how many Sarah ate.
Mrs. B: Can we find out how many the others ate?
Students: Yea!
G: Maria ate fifteen more than Sarah.
Mrs. B: How can we know how many Maria ate?
G: Because we know that Sarah ate twelve.
Mrs. B: So, we are comparing Maria to Sarah. What do we do?
O: Add fifteen to twelve.
Mrs. B: O.K. (making a table on the blackboard and adding fifteen to twelve under Maria's name) So, Maria ate twenty-seven.
Mrs. B: Now, what can we know from this?
N: Erica.
Mrs. B: Why Erica?
T: Because we just add twelve to twenty-seven.
Mrs. B: (adding and writing 39 under Erica's name) Why? (explaining) Because the problem says so.
T: (explaining) Erica ate twelve more pieces than Maria.
Mrs. B: And now, how do we go on?
O: Jennifer ate thirteen more pieces than Erica.
Mrs. B: (asking V) What do we have to do now?
V: Add thirteen to thirty-nine.

Mrs. B wrote 52 under Jennifer's name. Then she reflected on how the first answer, 25, was determined. Next, the class discussed the result and ways of getting it. Some students had done the problem in a different way.

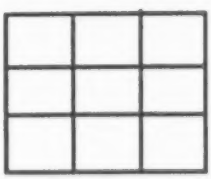
H: I added them all and got fifty-two!
Mrs. B: (going back to the solution written on the blackboard) What did we make?
C: A table.
Mrs. B: Thank you C.

Mrs. B observed that the students worked backwards. (The class had worked just once, early in the semester, on problems of this type.) She told H that, although he got the answer, it was important to know how he got it. What he did was all right, but sometimes problems have so much information that tables are needed. Mrs. B added that working backwards was probably the best way to do problems like the one just completed in class.

Excerpt Four (Problem Solving with Manipulatives). Mrs. B distributed unifix cubes among the students. They worked in three groups of six students each. The class was very excited. Mrs. B then asked the students to build an object that has a volume of seven and with the condition that each of the three groups' objects must be different. After building an object, each group went to the front of the class to show what they had done. The students seemed very proud of their work.

When all groups had finished, Mrs. B told them that there were more than ten different objects with a volume of seven. Next, she asked the students to work in groups of two or three and build objects with a volume of fifteen.

The groups that finished the last activity were asked to build a box with a volume of twenty-seven. Mrs. B directed, "It can't be just flat. It has to be high. The bottom layer has to look at least like this" (writing on the blackboard):



Five minutes later, when the class had to leave for lunch, half of the class had started and were still working on the three-dimensional box.

Students' Misconceptions

A second kind of problem-solving activity Mrs. B used was based on students' misconceptions regarding basic operations. As in other problem solving activities, emphasis was placed on the process rather than directly on the results of those operations. The whole class was involved in the discussions, and the excitement on the part of the students was visible. Expressions of amazement while they reflected on their misconceptions were a common sight. Mrs. B told me that the manipulatives to demonstrate misconceptions added considerably to the "kids' understanding."

One difficulty Mrs. B has noticed is in the students' ability to transfer learning of a topic from a representation based on manipulatives to an abstract one. As she observed, "Kids understand and visualize with the manipulatives. The trouble (for some of them) is when they transfer to paper." Subtraction problems involving regrouping reflected this difficulty. Following are some observations regarding the analysis of students' misconceptions about subtraction.

Excerpt Five (Misconceptions in Subtraction #1). Mrs. B lead into a discussion of

subtraction by commenting on the students' answers in a previous subtraction assignment and asked them why they got so many different answers for the same subtraction problems. She wrote a problem on the blackboard and then represented the minuend and subtrahend with base-ten blocks.

$$\begin{array}{r} 265 \\ -\,147 \\ \hline \end{array}$$

Mrs. B: I'll show you what some people did.

$$\begin{array}{r} {\scriptstyle 16\ 15} \\ 2\not{6}\not{5} \\ -\,147 \\ \hline \end{array}$$

Mrs. B: No trade was done. So, 275 (referring to what the minuend should be in order for their procedure to be O.K.).

A student started solving the subtraction by first crossing out the 5, writing 15 above it, and then writing a 5 above the 6:

$$\begin{array}{r} {\scriptstyle 5\ 15} \\ 2\,6\not{5} \\ -\,147 \\ \hline \end{array}$$

Mrs. B told this student that his work was wrong. He had to get one ten from the sixty **before** adding it to the 5 to get 15 units.

Some students regrouped when it was not necessary to do so.

$$\begin{array}{r} {\scriptstyle 1\ 15\ 15} \\ \not{2}\not{6}\not{5} \\ -\,147 \\ \hline \end{array}$$

Mrs. B observed that there was no need to add 10 tens to 5 tens when working on the tens column.

"I already have 5 (tens) and 5 is bigger than 4."

Excerpt Six (Misconceptions in Subtraction #2). In another observation, the class started with the students sitting on the floor near Mrs. B to work on a homework assignment. A white paper screen, a projector, and some base-ten blocks were ready to be used. The class discussed how some of the problems were solved. The conversation was frank. While Mrs. B commented on what the
students did, the hands of those who didn't understand Mrs. B's explanation quickly went up.

The first problem was the subtraction

```
 6 0 4
-2 7 8
```

Mrs. B showed what some of the students did:

```
 5  14
 6 0 4
-2 7 8
```

She said that one hundred can be regrouped as ten tens and not as one ten. The overhead projector was used to show the students the concrete representation of that situation. There were now five flats, ten longs, and four ones. On the blackboard there were three columns, separating the hundreds, the tens, and the ones. The problem was corrected to

```
 5 10
 6 0 4
-2 7 8
```

Next, Mrs. B asked the students: "What am I really doing when I cross out the ten over the zero? ... I trade one ten for ten ones." This was done first with the blocks and then with the written representation on the blackboard:

```
   9 14
 5 0
 6 0 4
-2 7 8
```

Mrs. B asked the students: "Can I do the problem?" The students answered "yes" in unison. Working together, the class arrived at the answer 326.

The next example was

```
  500
- 247
```

The students worked with the blocks and the projector.

Mrs. B: (writing down what some students did) Let me show you what some boys and girls did:

```
 10 10
 5 0 0
- 2 4 7
```

Students: Where do you get those 10s from?

Mrs. B: What you are giving me is a number much bigger than 500.... What do you have to do?

K: (a general answer for other subtraction problems) Mark out the 5?

Mrs. B: Not mark. What do you do on the projector screen? (turning to T) T, don't feel bad. I've seen many boys and girls doing this... What do I need to do?

T: Cross out the 5 and write a 4.

Mrs. B took away one flat on the projector.

T: Take this hundred and trade it in for 10 tens.

Mrs. B did it on the projector and wrote on the blackboard:

```
 4 10
 5 0 0
```

$$\begin{array}{r} \\ -247 \end{array}$$

Mrs. B: Can I do it? (asking the students if what they did was enough for them to do a direct subtraction) No?

L: Take one ten away, make it 10 ones.

Mrs. B did this with the blocks:

$$\begin{array}{r} 9\ 10 \\ 4\ \cancel{10} \\ \cancel{5}\cancel{0}0 \\ -247 \end{array}$$

Mrs. B: Do you think it will work?

Students: Yea!

Working with Mrs. B in the direct subtraction, the class arrived at the answer 253.

Excerpt Seven (Misconceptions in Subtraction #3). In the following example, the students worked on more examples associated with their misconceptions about subtraction. The next examples include other strategies that were used by them in order to get rid of conflicting situations. The first example was

$$\begin{array}{r} 400 \\ -235 \end{array}$$

Mrs. B: Some people would do this:

$$\begin{array}{r} 400 \\ -235 \\ 235 \end{array}$$

Mrs. B: What did they do to get an answer like that? They assumed we could change the numbers around (subtract the smaller digits from the bigger ones).

Mrs. B worked with the blocks on the front desk. The students were quiet. K went to the

blackboard. Mrs. B wanted the class to react to what he wrote. K wrote on the blackboard:

```
  4 10 10
- 2 3 5
  1 7 5
```

Although K did not change the hundreds digit, his answer 1 for the hundreds column was correct. It is possible that he did this mentally without bothering to write 3 in the place of 4.

Mrs. B: You pulled these ten units out of the air. You left ten tens without taking the ten units from one ten.

K seemed confused and asked, "What if we change the numbers?" Mrs. B showed K's mistake with an example that avoided the zero in the units column.

```
  4 0 3          3 10 13
- 2 3 5          ~~4 0 3~~
                - 2 3 5
```

She waited for some reaction. The class was quiet for a while, until a different suggestion was presented:

```
  2 0 0
- 4 3 5
```

Mrs. B Can you do it?
Students: (unanimously) No!

When they got the answer 165 to the original problem, Mrs. B compared it with the previous answer, 235. The students seemed amazed by such a discrepancy.

Excerpt Eight (Misconceptions in Subtraction #4). The last example shows a student regrouping without need for it. The problem is

```
  4 0 3
- 2 1 2
```

A went to the blackboard and couldn't do the problem.

Mrs. B: Is this problem different from the others?

A wrote down:

$$\begin{array}{r} {}^{9} \\ 3\not{0}\,{}^{10} \\ \not{1}03 \\ -\,212 \\ \hline 181 \end{array}$$

Another student pointed out to A that he could take 2 units away from 3 units without having to add 10 units to 3 units. Indeed, A did the subtraction but he might have felt that it was necessary to add the ten units anyway. A further discussion of the effect of A's procedure on the tens column did not come up. Instead, the problem was reworked leading to the right answer, 191. Mrs. B remarked that regrouping was not something the students had to do in every situation.

It was interesting to note Mrs. B's preference for the term "regrouping" instead of "borrowing", a term used by many people. From past experience Mrs. B had realized that for some students the term "borrowing" implied that, at some later point in time, the borrowed units would have to be returned to their original position. The term "regrouping" seemed especially appropriate as the students worked with manipulatives.

Mrs. B's preference for the use of "regrouping" contrasted in many cases with the parents' use of the term "borrowing." She observed that it was difficult for many students to deal with regrouping at school and borrowing at home. However, this conflict was successfully handled through meetings between Mrs. B and some parents.

Integrated activities

The third method through which Mrs. B presented mathematical content, can be characterized as an integrated approach. Mrs. B commented that mathematics is generally taught in isolation from other subjects, a perspective that should be modified. Although I was able to observe her using this approach in one situation, the two previously described methods, introducing concepts and problem-solving activities, appeared to be utilized more.

I had the opportunity to observe mathematics being taught integratively during Thanksgiving week. The whole class was excited, preparing for a school presentation of some of

the events that took place during the Mayflower's voyage. Mrs. B and her students worked on a concept map, displaying the things that were brought to the new continent.

Mathematics arose naturally from this situation. For instance, the students' curiosity to know the number of years since the arrival of the Mayflower got them engaged in the subtraction of 1620 from 1990. They were told that 102 passengers plus 30 crew members were on board the Mayflower, 46 pilgrims among them. The need for material sharing among those people led the class to a discussion of the number of homes needed when they arrived in the new country. Mrs. B informed them that approximately six people shared a home.

Also, the sharing of food among the settlers, such as allotting two bushels of peaches per household, led the students to examine mathematical situations. The number of peaches that any one person could eat, then, was determined by the household size.

Some information that was essential for the students' presentation was the length of the ship, 90 feet. The class was to compare that length with the length of the classroom. Mrs. B wanted to give them an idea of how little space those people had during their trip. Embedded in such an activity are the concepts of ratio and proportion.

Later on, during one of our interviews, Mrs. B commented that the concept of ratio and proportion could be naturally introduced through an integrative approach. She acknowledged the richness of such an approach and the effort it takes to provide the students with a strong mathematical background through its use. She planned to have her students work on an integrated activity during the December holiday break.

Mrs. B's Reflections About PETTM

This section contains Mrs. B's considerations about her teaching practices. It includes her reflections about such things as the textbook, the *Problem a Day* book, and the importance of teacher awareness of the influence of projects such as PETTM. Mrs. B also comments on her knowledge of the students and PETTM workshops. Included are her suggestions for broadening the dissemination of the project ideas and for new workshops.

Regarding the project, Mrs. B told me that although she already practiced a non-traditional approach, which included asking for students' explanations, considering problems with different answers, and not limiting the students to drill and practice, the project allowed her the opportunity to "formalize" her ideas. Mrs. B is very aware of what she is doing. The project helped her to consciously express much of the tacit comprehension she has of teaching practices: "Now, I write more down. I am conscious of that. I feel more comfortable with that."

One example of this refers to the textbook (*HBJ Mathematics Today, Level 3*). Before the project, Mrs. B followed the textbook without considering other options. Students would learn through practice of most if not all of the exercises in the textbook. Mrs. B commented that she

didn't know how to change such a practice. The project provided a means of change for which she was looking. Currently, the textbook is used only where practice is needed.

The resource book *A Problem A Day For Primary People: A Resource Book of Daily Process Problems for Students in Grades 1-3*, whose principal authors are Mrs. B and another teacher, was developed along with PETTM personnel during two summers. Mrs. B stated that the summer experiences were crucial. They involved "hands on" experiences in "choosing, organizing, and writing the problems in the book." The summer experiences allowed Mrs. B to gain self assurance and independence from textbooks. She is using, reviewing, and changing the resource book and feeling very comfortable with a more varied approach to teaching. "Now I can do it myself, generate problems and ideas depending on what I need." Mrs. B is also considering having the students write their own problems.

Tied to Mrs. B's current view of textbooks is the greater use of manipulatives in her class. Mrs. B employed manipulatives in the study of fractions and is planning to use them when introducing multiplication. She said that the introduction of a topic through manipulatives, followed by pictorial representations, provide a means for students to understand that topic much more clearly.

Mrs. B's teaching approach has changed from the use of paper and pencil activities to the use of manipulatives. On a number of occasions I observed positive student reactions to manipulative activities and it is apparent that Mrs. B. has been successful in achieving a balance between concrete and more abstract forms of mathematics.

Another example of change in Mrs. B's teaching practices as a result of PETTM has to do with the amount of time she spends planning mathematics lessons. She told me that at first, despite her ideas, it took her a lot of time to decide upon what to do in the classroom. Now she is able to go through a book much faster, knowing exactly what she wants to do. She can build problems for her students in accordance with their needs. Introductory lessons are generally related to the students' everyday lives.

Also, Mrs. B's practice regarding questioning and group work have changed. Before the project started, there was little group work and class discussion about mathematics. Students would work individually and she would ask individual questions of the students. Now, there is more group work on mathematical problems. Most problems are followed by class discussions which frequently include groups reacting to other groups' solutions.

Mrs. B considers her teaching skills as something that she has now internalized. As she expresses it: "Now, [teaching] is a part of me. It's so automatic that I can build stories while I'm doing something else." And, related to good problems and questions for students, Mrs. B states "The problems come with [practice]. They generate themselves."

As noted, problem-solving activities and an integrated perspective reflect much of Mrs. B's

view of mathematics education. Her current practice of observing and working on the ways of representing and explaining solutions to mathematical problems allows her to get a good feeling for the students' level of conceptual understanding. Mrs. B observed that, besides working on the understanding of concepts, it is important for the students to practice procedures. While the former is dealt with in the classroom, the latter is generally left as a homework assignment. During class time, the students usually work on drill and practice that is embedded in the solution of broader problems. In those cases, such practice is only a secondary aspect of instruction; conceptual understanding is the major aspect.

Both practice exercises and problem-solving performance are considered by Mrs. B in her evaluation of students. Three or four times a week the students work on homework assignments, practicing what they have learned in school. Once a week they work on a similar assignment in the classroom. Those homework and classroom assignments are graded, but only for Mrs. B's own information about the students. The assignments are then analyzed by the students, who discuss them in small groups. In general, students seem comfortable enough to show mistakes to their colleagues. Although assignments contribute to her overall impression of a student, Mrs. B does a lot of evaluating while teaching: "I do it mentally. It is just part of my teaching."

I asked Mrs. B if it was necessary for a teacher to be aware of her or his own practices in order to openly accept the influence of projects such as PETTM. She answered that not every teacher who participated in the project was "aware" or felt "frustrated" about what went on in her or his classroom. However, she felt that awareness and frustration were strong factors in influencing a teacher's decision to engage in projects of that kind, especially those projects which involved the cooperation of both primary school and university personnel, as is the case with PETTM. Mrs. B mentioned that professor John LeBlanc, the director of PETTM at I.U., always made it clear that the teachers had to think about and do things they were interested in and that the I.U. personnel involved in the project would support teachers with any directions they chose with respect to teaching mathematics.

Mrs. B enjoyed most of the workshops and regretted the fact that she didn't have enough time to reflect on all of the ideas and materials presented. She thought that it was helpful for teachers to engage in discussion sessions on new ideas about teaching.

Mrs. B thought that the teachers who got the most support from the I.U. personnel should take the responsibility to share the project's ideas with other teachers. She has been able to see other teachers using the resource book she coauthored. She notes that many other teachers are uncomfortable in going ahead and designing their own problems in the way that she does. She suggested that some workshops should be included that focus on the activity of writing problem-solving books.

Mrs. B felt very comfortable in asking people from I.U. for help. Now, she knows what

she wants to do and her questions relate to possible ways of doing it: the how to. As she observed, "There are situations where I'm stuck. But I know more clearly what I want." Mrs. B's confidence and openness were certainly associated with her close ties to PETTM staff at I.U. Her suggestions bring to attention the fact that such ties should be further nurtured between teachers and other professionals.

Conclusion

The observations and interviews discussed herein were intended as a description of Mrs. B's current teaching practices and how they are tied to PETTM. One main aspect of growth as a teacher is the change from a tacit comprehension of teaching to a conscious one. This is an important aspect since it allows Mrs. B to be more attentive to the characteristics of both her needs and those of her students. It is also relevant to mention that the changes in Mrs. B's teaching practices have been accompanied by a state of increased confidence. Although she agrees that no one can ever be a "perfect" teacher, she is more comfortable now with her teaching skills than she was before PETTM.

Mrs. B employs a variety of methods in her classroom. They can be described as **the introduction of concepts, problem solving,** and **integrated activities.** Problem solving can be divided into two categories: one in which the students solve problems and another in which they analyze their conceptual misunderstandings. In all her teaching, Mrs. B strives for independence in her students. She wants every student to develop confidence in mathematics. Mrs. B is primarily concerned with process-type questions and this pattern is also emerging in discussions among the students who at present still favor product-type questions.

By being one of the authors of the resource book, Mrs. B went through what she feels is an essential experience in the life of an educator: authorship. She is very confident about her abilities regarding that work. And she feels free enough to seek approaches that can be meaningful to her students.

Besides daily use of the resource book and increasing independence from the textbook, Mrs. B is making extensive use of concrete materials. It is worth noting, however, that such material is not considered as an end in itself; the transition from manipulatives to paper and pencil activities is viewed by Mrs. B as essential.

The influence of PETTM on Mrs. B's teaching practices has been very positive. The workshops, discussions, and involvement in the development of a book have contributed to the process of her becoming a teacher and researcher. Furthermore, the support of the I.U. personnel was fundamental.

Mrs. B expressed some concerns about the teachers in the community. Some of the teachers that have been involved in PETTM are already in the process of sharing their experience

with other teachers. The participation of I.U. personnel in such a process is still essential when one acknowledges that the main purpose of PETTM is to build a continuous dialogue in experiencing education.

Mrs. M: A Fifth Grade Teacher

Mrs. M was one of the fifth grade teachers at Highland Park Elementary. As part of the evaluation of the PETTM project, Mrs. M was observed and interviewed during the months of March and April, 1991. During that time, Mrs. M was observed five times and interviewed several others.

The body of this report is divided into four sections. In the first section, I describe Mrs. M's classroom environment: its physical arrangement and human atmosphere. The second section is focused on the characteristics of the lessons that were developed during my stay in Mrs. M's classroom. Considerations about the content of those lessons as well as the approaches used to teach the contents are also provided in that section. In the third section, I focus on group work, the process of evaluation, and the most frequently used textbooks. The fourth section of this report reflects most of the conversations in which Mrs. M and I engaged. That section includes some of Mrs. M's personal feelings and reflections as a teacher.

Mrs. M's Classroom Environment

During the periods in which I went to Highland Park Elementary to observe Mrs. M's classroom, she had thirty students. Like Mrs. K, the second grade teacher, Mrs. M had an unusually difficult group of students to work with. The seats in Mrs. M's classroom were arranged to form a large rectangle which was almost the size of the room. With the exception of the students who sat at the corners, each student was located between two of her or his colleagues. This formation made it difficult for students to move around the room. There were, however, times when the students would leave this formation. This happened when the students worked in groups, presented and discussed solutions of problems, or simply walked around to talk to their colleagues.

I felt that the atmosphere in Mrs. M's classroom was very relaxed. Mrs. M and her students developed a very close relationship. She was always very patient and transmitted a feeling of tranquility to the class. In one interview, Mrs. M told me that her students were very cheerful, a characteristic that restrained her from being too serious with them. The students seemed to appreciate the relaxed atmosphere and cooperated with Mrs. M in various settings including individual work, group work, and whole class discussion. When the conversations were too loud or when a colleague was interrupted, Mrs. M discussed the inappropriate behavior with the class. Her tone was such that students took her comments seriously.

The Lessons

Most of the lessons I observed in Mrs. M's classroom were introductory geometry lessons.

There were two occasions in which the class was involved in problem-solving activities. In what follows, I describe and provide examples of both the geometry and problem solving lessons. To paint a clearer picture of the lessons, the descriptions also include comments from my conversations with Mrs. M as well as my personal reflections on the situations.

Introductory Lessons

Mrs. M's introductory lessons were characterized by two distinct phases. In the first phase, Mrs. M had a conversation with her students about the topic to be introduced. In general, those conversations evolved with the full participation of the students, who presented their ideas about the new concepts. Mrs. M was very discrete, seldom intervening in situations where the students asked for clarification. However, when a group or the entire class seemed unable to proceed, she provided hints to get students working again.

In the second phase the students worked, individually or in groups, on assignments that were closely related to the concepts introduced earlier. Those assignments involved the students in exercises, which consisted mainly of recognition and representation of concepts, and in the creation of geometric designs.

The following two excerpts exemplify typical introductory lessons in Mrs. M's classroom. In the first excerpt, the topic of the lesson was symmetry. The second excerpt shows how the students where introduced to the vocabulary and symbolic representation of lines, rays, line segments and angles.

First excerpt, first phase: Because geometry was being addressed for the first time in the year, Mrs. M decided to start the class by asking her students about their knowledge of geometry. Some students answered that they learned some notions of geometry in the third and fourth grades:

P:	Lines and circles.
T:	I don't know the words ... perimeter, area.
Mrs. M:	You probably learned geometry before you went to school. Remember when you talked about shapes?
A:	A pentagon.
Mrs. M:	How many sides [does a pentagon have]?
A:	Six.
Mrs. M:	Are you guessing?
A:	Five.
Mrs. M:	O.K.

Other answers were: "Calculating the area of a figure by breaking it in small squares," "Geometry is something you learn in college," or "Geometry is a hard stuff."

Next, Mrs. M drew some polygons and a circle on the blackboard. She showed the students three dimensional objects. Mrs. M showed a basketball. The reaction of her students was fast: "a sphere." She explained: "It's not solid. Air makes it look like that. But it's a sphere." Next, Mrs. M showed a cup to which the students answered: "a cylinder." Mrs. M told the students that the cup was filled inside. After that, Mrs. M drew a cone and the students shouted: "a cone." Finally, Mrs. M showed her students a cube, telling them that the cube was "based" on the square.

After this initial conversation about geometry, Mrs. M started to focus on symmetry, the topic of the day. She asked her students whether they had already heard about symmetry. The students expressed their ideas in the following ways:

S: It is like as if you cut something in half and both parts look the same.

Another student stood up and showed her colleagues the symmetric drawings in her sweater. Other students followed with their examples:

B: The screen for the transparencies.
Mrs. M: In how many ways can you cut the screen and have look alike parts?
B: In four ways.

That student then showed on the blackboard what it would look like with the four cuts.

Another example was provided by Mrs. M, who told the students that our own bodies could also be considered a symmetric shape.

The class continued with a slide presentation on symmetric shapes. Before Mrs. M started showing the slides, she asked the students to focus their attention on the lines of symmetry that

were present in those shapes. The students looked amazed while Mrs. M showed them slides of flowers such as the trillium, the pansy, the morning glory, the orchid and the maple. For each slide shown discussion focused on lines of symmetry and the beauty of the flowers. Slides showing a butterfly, a sand dollar, fruit, snowflakes, and so forth followed with the same kind of discussion and student amazement.

First excerpt, second phase. When discussion of the slides ended, students worked on folding paper and creating lines of symmetry. They used colored pencils in their designs and created appealing pictures which also demonstrated symmetry. The next day, the class started with Mrs. M mentioning how appreciative she was of the designs and pictures that the students made while working on symmetry. Those pictures hung on a poster and the students discussed the symmetric features of each one. The fluent discussion that followed transmitted a warm feeling in the classroom. It was an enjoyable moment for the students.

Second excerpt: first phase. Mrs. M drew a line segment, a ray, and a straight line on the blackboard. Then, she asked the students which of the drawings they thought would go on forever in one direction. T indicated the ray as his answer, to which his colleagues agreed. Mrs. M, then, introduced the name "ray" and a representation for rays to the class. The names and representations of lines and line segments were introduced similarly.

Next, the students were asked to find some "line segments" in the classroom. One student showed her pencil; another pointed to the tiles in the ceiling, referring to their borders. When the class was looking for examples of "rays," Mrs. M provided one: she identified the sun light as a collection of rays.

The next concept to which Mrs. M's students were introduced was that of the angle. Mrs. M drew a right angle on the blackboard and asked the students to look for examples of such angles. They pointed to the corners of the door and the floor. Mrs. M then told the students that those angles were called "right" angles.

Mrs. M then drew a circle and told the students that it measured 360°. Next, she drew a line through the center of that circle and asked the students what the measure of half a circle was. The students answered: "180." Then, Mrs. M drew a perpendicular to the previous line through the center of the circle, asking how many right angles there were in the circle and what the measure of each of those angles was. The students answered: "four" and "90," respectively. After the introduction of right angle, the class worked with acute and obtuse angles. In this part of the lesson, Mrs. M was very direct. She used the blackboard to define those angles for her students.

The introduction of the fact that the central angle of a circle measures 360° did not appear to sit well with some students. It was interesting to observe a student (K) questioning Mrs. M about

how she knew that a circle measured 360°. Mrs. M didn't want to go into the details of such a question and told K that she knew that answer because her fifth-grade teacher asked her to accept it as such and that's what she did. She added to that: "Now, I'm asking you to do the same. Okay?" Mrs. M told me that in later lessons students used protractors and compasses to measure angles and construct circles. Discussions in these later lessons involved the previously mentioned incident of acceptance without proof, clarifying K's question.

Apart from having looked for a few examples, Mrs. M's students did not draw or use the geoboard or any other concrete material to experience the concepts to which they were introduced during the first part of this lesson. After the initial conversation was over, however, they started to work with geoboards under Mrs. M's guidance. First, all the students constructed right angles with their geobands. Then, Mrs. M asked them to construct other angles, using their right angles as a reference. She demonstrated the technique by using a right angle that was drawn on the blackboard. Mrs. M put her finger over point **B**, the vertex of her right angle, and drew an acute angle keeping **B** as the vertex of the latter. Then, she asked the students to make equivalent acute angles on their geoboards.

While the students worked on the construction of the acute angle, Mrs. M passed by them checking what they did. Most students took a long time to hold the vertex of their right angles and construct the new angle. When they were asked to construct an obtuse angle in a similar way, most of the students understood more quickly.

Second excerpt: second phase. The following day, the students worked on some assignments which were related to the topics that had just been introduced to them. In their first geometry assignment from the text *(HBJ Mathematics Today*, Grade 5, 1985, pp. 117-118), the students had to name and classify certain given angles as right, acute, and obtuse. Mrs. M did the first exercise on the blackboard. Then, she walked around the students and talked to them individually. The students worked diligently on the assignment.

For the second assignment, each student was given a geoboard, five geobands, and a worksheet from the book *Hands On: Geoboards, Book 1* (1986), p. 17. They had to associate the pins that were drawn on the worksheet with pins on the geoboard, connecting the latter with one geoband. Next, they had to copy on the worksheet the shape they got on the geoboard. As in the previous assignment, Mrs. M did the first example on the blackboard and then went to observe her students. She praised their accomplishments and provided support for those who needed help.

The second geoboard assignment from *Hands On: Geoboards, Book 1* (1986), p. 18 was more creative. This time the students had to create their own shapes and copy them on the worksheet. The class was very involved in this assignment which combined geometry and art. They shared their creations with each other as they were spontaneously led to work together in

groups of three or four. I often observed students asking and then receiving suggestions from their colleagues.

In brief, new topics were introduced apart from specific applications or problems. I did not observe one situation in which a new concept was introduced via a problem-solving activity. Regarding this observation, Mrs. M told me that it was difficult for her to introduce some of those concepts, especially those considered in the second excerpt. She said that she needed more information on how to deal with the introduction of such concepts.

Problem-Solving Activities

Two problem-solving activities were observed during my study of Mrs. M's class at Highland Park Elementary. Most of the students got very excited about working on these activities. The first problem, in particular, generated a lot of enthusiastic discussion.

First excerpt. The first activity was called "The Hidden Shape" from the book *Mastermind* (1985), p.34 and consisted of a collection of problems. In the first four problems the students were given a large square drawn on a sheet of paper. They were asked to count the number of triangles and blank squares that were inside the large square. In the fifth problem, the students were asked to draw a large square with five blank squares and eight triangles inside it. Mrs. M asked the class if what A presented could be considered as a solution for the problem. Some students felt that A's solution was correct while others were not sure about their answers. Then, Mrs. M suggested that the class count the number of squares and triangles inside the large square. After the counting was done, all the students accepted A's solution as valid.

After A's solution was presented, Mrs. M asked the class to work on the problem dividing the large square into small squares using parallel and vertical lines. Three solutions where presented in accordance with the new condition. After the third solution was presented, a girl noticed that there were more than eight triangles inside the large square. She went to the blackboard and pointed out an additional triangle to the class.

This activity got the attention and excitement of the whole class. Every student wanted to present his or her solution to the class. After the solutions above were presented and discussed, the students noticed that all "additional" solutions were similar to one of those already presented.

Second excerpt. Mrs. M presented the following logic problem to the class. (The problem is the "Skateboard Collision Problem" from the book *A Logical Mystery Tour* developed by PETTM.)

Steve, Ted, Rolando and Arturo collided while skateboarding. After their collision, they

noticed that all their skateboards and helmets were mixed up. Figure out which items belong to each child.

Clues:
1. Steve's skateboard and helmet are the same color.
2. The person with the hot pink skateboard wears an electric blue helmet.
3. Rolando has a neon green skateboard.
4. Rolando's helmet is the same color as Arturo's skateboard.
5. Arturo's skateboard would match Pink Panther's color.
6. Steve does not wear a neon green helmet.
7. Steve either owns the electric blue or tangerine orange skateboard.

(A matrix follows with the children's names written as row headings and the four different colors of skateboards and helmets written as column headings.)

Mrs. M asked the students to read each clue while she wrote them down on the blackboard where a matrix had already been drawn. She marked, with an X, the places in the matrix which were eliminated by the children. For instance, when the children read clue number 3 they eliminated the places available for the neon green objects in Steve's row due to clue number 1.

The class filled in the matrix according to the following sequence of clues: 3, 5, 4 and 2. After a few more minutes, some girls decided about the items belonging to Steve: "their color is tangerine." When asked by Mrs. M to explain their decision, the girls told her that clue number six, added to what they already knew, was all they needed to find the answer. Next, the students finished reading the clues and completed the matrix for Ted's items.

This problem was easy for some of the students. The girls mentioned in the previous paragraph didn't even bother reading clue number seven. In fact, the problem could be solved having some of its clues eliminated such as clues numbered six and seven. On the other hand, for most of the students the problem solution was developed too fast. Some students were only copying the answers and did not really understand why the answers they were copying made sense. Three weeks later Mrs. M told me that she was aware that the problem solution had moved too quickly for some students. She hadn't had the time to develop the problem the way she would have liked to because there were other activities to be completed that day. She also told me that a few days after the Skateboard Collision problem was discussed she had the students involved in a similar problem. On that occasion, they worked in small groups, which helped the students who hadn't understood the Skateboard Collision problem.

Mrs. M's primary conception of problem solving in mathematics is closely associated with mathematical applications. First, the students learn the mathematics needed for use in some project

of interest. Next, they apply that mathematics in the development of the project. When the project is completed the students report the project's results. Mrs. M was planning to have her students work on a string-art project as part of a problem-solving activity. This activity involved students in dealing with geometrical applications as a means of developing problem-solving skills.

In one of my discussions with Mrs. M, she mentioned a particular project on interpreting graphs where the students had to prepare and conduct a survey. In her view, interpreting graphs is a concept repeated in most fifth-grade subjects (i.e., mathematics, science, social studies, reading, and language). Mrs. M engaged students in a project that integrated these subjects. By generating, collecting, analyzing, and recording data, students' understanding is enhanced.

Mrs. M told me that after her experience with PETTM she has extended her students' learning by presenting more applications of mathematical topics being studied. For example, to extend her student's learning in geometry, Mrs. M was planning to take them for a walk so that they could associate what they did in the classroom with the world outside it.

During most of the excerpts in this report Mrs. M provided her students with examples of what they had to do, especially for their assignments. When asked about this practice, Mrs. M agreed with my observation, explaining that the practice was intended to provide her students with a better sense of what they were supposed to do in their assignments. Mrs. M's perceived that her students were "average" and felt that this practice was appropriate for such students.

Mrs. M also perceived her students as willing to work hard but not too hard, especially on long assignments. She felt that one of her main jobs was "to help the lower level students to catch up [with the rest of the group]." After providing examples of certain problems, she would generally observe what each student was doing, encouraging those who did not start and complimenting those who were already engaged in the activity.

Group Work, Evaluation, and Texts

Students in Mrs. M's class often worked collaboratively although Mrs. M usually left it up to students in the class to decide when and with whom they wished to work. My observations indicated that she neither formed groups nor intervened when several students wanted to form one on their own. I asked her if this was the norm in her classroom and she indicated that it was, explaining that the process of grouping was very unstructured. However, during the development of projects, for instance, she stated that she did provide structure, organizing the groups according to the knowledge she had of her students.

Mrs. M evaluated her students according to all the activities in which they were involved. When grading classroom and homework assignments, Mrs. M said she looked at both correct and incorrect answers and took into consideration the amount of student effort in completing the assignment. For students whose answers were incorrect, effort was a major consideration in the

grading process.

When students in Mrs. M's class worked on lengthy projects, such as the previously mentioned survey project, a different evaluation approach was used. In such a case, Mrs. M considered each of the tasks students had to complete as part of the project. These tasks included the initial organization of the project, its actual development, and the preparation of the final report.

A very important aspect of Mrs. M's evaluation process is student participation in classroom activities. She wants to have all the students debating the topic under discussion. Something that I observed in my visits to Mrs. M's classroom was that, in her effort to encourage student participation and risk-taking, she did not place importance on right answers. Regarding student participation, Mrs. M said that she sometimes felt frustrated with the fact that, generally, it was the same people who provided answers and explanations. Many times, she asked those people to keep themselves from answering so that others, who had more difficulty in doing so, could also contribute to the growth of the group. During my visits to Mrs. M's classroom, I noticed that she allowed a reasonably long wait time (15-20 seconds) for the students to express themselves.

Mrs. M had been using the textbook only for specific situations such as practice work. She has incorporated more hands on and concrete activities in her math classes. As a result of the confidence in teaching mathematics she has gained from the project she is now comfortable using the book as a tool rather than a guide. For example, she stated that "I'm changing the order of things in the book. I'm not as intimidated as before."

When I asked Mrs. M about the resource books (*Moving on with Geoboards* from Creative Publications, *The Problem Solver 6* from Creative Publications, and *A Logical Mystery Tour*), she complained about the lack of geometric situations in them and mentioned the need to address such a limitation as soon as possible. She appreciated Jackie's support and contribution in getting interesting problems.

Further Conversations with Mrs. M

When Mrs. M first began working with staff from PETTM, she thought that she already knew most of the things a teacher should know. She changed her mind some months later. At a personal level, PETTM has changed many of Mrs. M's previous conceptions about being a teacher of mathematics. She told me that she "was really happy about being part of it [PETTM]."

Before the project started, Mrs. M used to "beat the students till exhaustion for both [the students and herself]." She used a behaviorist approach to train her students to memorize the lessons and be able to repeat them without any signs of indecision. Mrs. M's involvement in PETTM helped her to modify that educational perspective. She felt that a broader view of mathematics education allowed her to do a better job as a teacher.

Mrs. M was very proud of her involvement in PETTM. During this past summer, she trained teachers from other schools that were not involved in the project. She also coached those teachers during the school year. As one culminating activity, Mrs. M and other colleagues made a presentation of their work at the 1990 meeting of the Indiana Council of Teachers of Mathematics (ICTM) in Indianapolis.

One of Mrs. M's greatest concerns is the ending of PETTM, at least in the formal sense. She asked if the teachers involved in the project would be able to continue sharing their needs and expectations with the I.U. personnel. She indicated that the I.U. personnel had always been very supportive to those teachers, especially in helping them learn to teach problem solving.

Mrs. M's involvement in PETTM has also increased her level of involvement with inservice for teachers in other schools and districts. However, such commitments have reduced somewhat the time she has available to work in her own classroom and on her own personal improvement. If on one hand all the new activities in which Mrs. M is currently involved contribute to her personal growth as an educator, on the other hand, the time she needs to meaningfully integrate what she learned from PETTM has been shortened. As she told me "I don't need necessarily to write new things. What I would like to do is to work on the things that I already have."

Mrs. M's last comments are tied to my observation that she developed little bits of content in each of her classes. For instance, in the case of introductory lessons on geometry, she discussed the idea and symbolic language of symmetry, lines and angles, parallel and perpendicular lines in just a few class periods. Mrs. M explained to me that the idea in the fifth grade was to have the students get a feeling from all those topics and that they would review them more fully in the sixth grade: "This is just a stimulation for them. They will recognize these topics when they see them again." Mrs. M stated that the practice of briefly introducing concepts presented a continual dilemma for her. On one hand, such introductions did seem to spark interest in some students. On the other hand, Mrs. M was hesitant about presenting concepts that she did not have time to fully explain.

Now, that the project is ending, Mrs. M is reflecting about what the main objective of the project was. Was it to change some people? The school? Or something else? Mrs. M told me that she was not sure if the project had changed the school or even if this was one of its main objective. Although the answers to these questions were not clear to Mrs. M, the fact that she was asking them indicated to me that she had a deep understanding of the complex forces which influence elementary school instruction.

Mrs. M's reflections led her to talk about the lack of internal meetings during the school year: "Everybody is very busy." Also, she felt isolated with respect to grade level since she was the only fifth-grade teacher in the school with the experience of having participated actively in the

project. Mrs. M would like to have people discuss and reflect on what was happening in classrooms: "Today, there is too much running and no time for reflection."

Conclusion

The description of my observations and interviews has been presented as evidence that PETTM has been influential on Mrs. M's approach toward education and teaching. Mrs. M's substantial skill as a mathematics teacher was apparent during the observations. Her interview comments confirmed this, and made it possible to identify some of the things she gained from participation in PETTM.

Mrs. M's former behaviorist/rote memorization approach to teaching has been supplanted by a model of interaction that encourages communication and participation. Mrs. M is a teacher who supports group work and individual effort. She evaluates her students by taking into account all the activities in which they are engaged. The involvement of her students in the process of answering a question or solving a problem is valued as much as the presentation of correct answers or solutions. Mrs. M generally allows adequate wait time for student response to questions.

Two types of lessons were observed in Mrs. M's classroom: introductory lessons and problem-solving activities. Introductory lessons were characterized by two distinct phases. In the first of these, the teacher and students had a conversation about the topic of study. Generally, new concepts were introduced through brief lectures or examples rather than via the presentation, development, and discussion of problems. The second phase in an introductory lesson consisted of the application and practice of the new topic.

Mrs. M associates problem solving primarily with mathematical applications. The central aspect of the applications is the development of projects inside or outside the classroom. However, as the excerpts reported here have indicated, Mrs. M sees problem solving permeating much if not all mathematics teaching and learning.

In summary, Mrs. M's involvement in PETTM has increased her level of responsibility to her students and to the teaching profession. Her perception of the school as a complex system of human interrelationships has provided her inharmonious ideas about teaching: a struggle between teaching for understanding and the meaningful integration of topics and teaching for the completion of a schedule of contents determined a priori.

Mrs. J: A Sixth Grade Teacher

This section of the report contains a summary of six observations and three interviews with Mrs. J, a sixth-grade teacher. The interviews took place during the spring of 1990. Mrs. J was a participant in the PETTM project during the 1988-89 and 1989-90 school years and was involved in writing materials for the project during the summer of 1990. Personal factors resulted in Mrs. J moving to a different school in the fall of 1990 and thus, unlike the other teachers observed, was a participant in PETTM for only two years.

The main body of this report is divided into five sections. In the first section, I provide a brief introduction of Mrs. J's perspectives regarding mathematics education. The second section is focused on Mrs. J's view of major mathematics topics for her sixth-grade class. A description of the methods employed by Mrs. J is provided in section three. Section four contains her thoughts on topics such as homework, evaluation, group work and calculators. A concluding section finishes the report.

Introducing Mrs. J

Mrs. J started teaching in the 87-88 school year at a school located one hour away from Highland Park. She taught remedial second and fifth grade students, following an individually guided education program. In the following school year she came to Highland Park and started working in the sixth grade. Her first year at Highland Park was also the first year of PETTM at Indiana University.

Mrs. J felt that she gained a number of new perspectives on teaching mathematics during the first year that she was involved in the program. The project contributed to the development of her self-confidence and provided a variety of useful resources. Mrs. J was amazed by the existence of so many problem-solving strategies and the idea that there were meaningful ways of teaching mathematics. Today she feels extremely comfortable with the sixth-grade curriculum and has no reservations about deviating from the textbook when she thinks it will help her students.

From Mrs. J's perspective, the primary reason for learning mathematics is to deal with problems in real life situations. Jobs, in particular, require mathematical skills and students should master those skills. In her words, the study of mathematics consists of "learning how to solve everyday problems and making sense out of the world. It depends on what your job is." She told me that her class visited a grocery store, invited a salesman for a talk, and had some I.U. students develop small business projects. In one of my visits, I observed her students talking with preservice teachers (PSTs) from the early field experience program about topics such as the organization of business enterprises.

Mrs. J views about what to teach were closely aligned with her views about the mathematics students would need once they finished school. Problem solving was a very important topic for

Mrs. J, and she noted that students should be assigned world problems that are hard enough to make them think. According to Mrs. J, the students have to learn how to work logically and have good computational skills. Referring to problem solving in real life situations, Mrs. J wanted her students to be "decision makers" rather than just "algorithm solvers." As she emphasized, "I want them to have strategies in their personality at the end of the year. Just do it in real world situations. I don't want them to just do operations. I want them to think," and raising her hands above her head, "[achieve] high levels of understanding." A project that students in Mrs. J's class were working on was to prepare a book of word problems. Each student was writing a problem and accompanying illustration for the book. Mrs. J brought this to my attention, observing that "if you can write a problem then you can understand it."

Most of Mrs. J's beliefs about mathematics were associated with her profession as a teacher. In the next three sections I provide the reader with a closer look into those beliefs.

Mrs. J's View of the Major Mathematics Topics for the Sixth Grade

The first topic covered by Mrs. J during the year of this study was geometry. There were many reasons for this, including the development of a learning atmosphere in which students shared ideas and also treated each other with respect. Much of Mrs. J's time during the first semester was spent dealing with behavioral matters, helping students feel success, and instilling confidence. Mrs. J told her students what she expected of them and the class was conducted under tight control. According to Mrs. J, that control had gradually been loosened, a tendency I could observe during my visits.

One of Mrs. J' major educational goals had been to increase students' confidence in their mathematical ability. In her view, "mathematics ability is very much influenced by confidence." To her, "every student can learn math," and, as she always said to her students, "you are the best sixth grade math class in Bloomington." The difficulties teachers faced lie in the fact that "they [the students] don't [all] learn the same way. This is the challenge."

The teaching of geometry through a series of activities involving paper cutting, paper folding, and mirrors seemed to Mrs. J to be a good way to get students involved in mathematics at the start of the school year. Moreover, the chapter on geometry was in the back of the textbook. Thus Mrs. J decided to involve the students in some hands-on geometry activities that were not a part of the regular curriculum. Mrs. J's decision was supported by Jackie Gorman, the mathematics resource teacher provided by PETTM, and by Monica Cougan, a mathematics education graduate student who had also been involved with activities at the school.

In addition to geometry, Mrs. J spent time during the first semester working on computational estimation (four operations), decimal computation (four operations), and mental arithmetic. While she felt that it was necessary to cover computational skills, Mrs. J considered

estimation to be a very important topic. She expected students to estimate several different ways on the same problem. Regarding mental math, Mrs. J told me that her class worked on it once a week. The topics emphasized were of the computational type, with the students learning how to combine numbers to operate with them. They had also developed strategies for the zero. Mental math had been applied to grocery shopping and the study of careers and was missing from areas such as geometry and measurement.

Later in the semester, topics associated with careers, salaries, and operating with money were developed in a project called "life project." The class also spent time working on problem-solving strategies and on the development of their problem-solving book. Mrs. J planned to spend the time left in the school year working on fractions, ratio and proportion, measurement, and a project involving the construction of bridges (see the School Component Chapter). At the time of my observations, the class was studying fractions with the assistance of Jackie Gorman (the mathematics resource teacher), who had introduced materials such as pattern blocks and Cuisenaire rods.

The texts used for mental math and problem-solving activities were, respectively, *Mental Math in the Middle Grades* by Hope, Reys and Reys (1985); and *Problem-Solving Experiences in Mathematics (Grade 6)* by Charles, Mason, and Garner(1985). The textbook *Mathematics Today (Level 6)*, published by Harcourt Brace Jovanovich (1985), had generally been used as a content guide. In our conversation about the course content, Mrs. J expressed her concerns in getting more teachers involved in discussions about what should be covered. She felt very strongly about this issue, observing that "the decision to cover those topics [fractions, ratio and proportion, measurement, and the bridge project] is mine." On the other hand, Mrs. J perceived the sharing of experiences with other teachers as a means of support most needed in periods of change and maturation. She said that discovering things by herself was scary sometimes. The presence of Jackie was fundamental in the building of Mrs. J' confidence.

Description of Mrs. J's Teaching Methods

Mrs. J was a very organized teacher. The time spent at the beginning of the year teaching students how to interact with the teacher and among themselves seemed to have paid off. During my observations, I could not find any gaps between activities. The students were continuously involved in activities in a way that differed from just being kept busy. They were either interacting with their classmates or working individually. Mrs. J told me that sometimes she felt a need for changes in the daily sequence. She used the term intuition to express her experience in "moving things around." For instance, some days she would have the students reading, talking, and doing some individual work. Activities would follow a sequence where students relaxed, worked quietly, relaxed again, worked in groups, etc., to avoid the monotony of an overly structured

learning experience.

During my visits to Highland Park Elementary I observed mathematics related activities being developed in several ways. A traditional approach was used in the development of what Mrs. J called a typical math class period. This approach differed entirely from the one utilized during problem-solving oriented activities. An integrated approach was used to tie mathematics with other subject areas.

The Traditional Approach

When Mrs. J taught using what she called the "traditional" approach, she verbally introduced a topic and showed some examples on the chalkboard. After the presentation, the students worked on exercises while Mrs. J did them on the board. Generally, twenty minutes passed until the exercises were compared. Finally, the topic was discussed by the whole class. If some students did not understand the lesson, which Mrs. J considered to be the "teacher's fault," she provided extra explanations on an individual basis or in groups. When the class was divided into groups, there were generally one or two students per group who grasped the topic. These students were instructed to share their ideas with their classmates.

Mrs. J's "traditional" classes took 45 to 60 minutes, depending on the difficulties found by the students. I had the opportunity to observe one of these classes. The students were introduced to the division of a decimal by another decimal after having worked on dividing decimals by whole numbers. An example (30.87/4.9) was presented by Mrs. J and one student immediately asked: "Can you round 4.9 to 5?" Mrs. J responded with "good guess, but..." Another student told the teacher to move the decimal point in both numbers one digit to the right to make 4.9 a whole number (49), "Then you divide like you did 'till today (308.7/49)."

As I observed, I saw no evidence that students understood the idea behind the rule. The approach used for this topic was the traditional rule-oriented approach. No materials were used. Neither was there a questioning of the type: 'What could we do to transform the divisor into a whole number? If the answer was 'multiply both numbers by 10,' the teacher could ask 'Why?' and so on.

After the rule was 'imposed' on the students it was time for practice. Some other issues were not addressed, such as 'the idea of adding a zero' in 30.87/.549 when transforming the divisor into a whole number. There was a great deal of verbal encouragement during the development of the new topic. While the students were getting the right answers, Mrs. J constantly encouraged them to work more quickly, saying things like "Let's do it in one shot so that we can move."

In our conversations, Mrs. J mentioned that she felt less than adequate when dealing with certain topics in mathematics. She seemed very comfortable talking about her limitations and was eager to improve that situation. Having observed the traditional approach utilized in the division of

decimals, as mentioned previously, I asked her to show me how she might do an initial introduction to decimals. She opened the textbook [HBJ] to page 208, where there was an introduction to writing tenths or hundredths as decimals and told me that, when possible, she would start the topic with a real life situation. This would serve as a "hook" for the students. Regarding materials, she told me that base ten blocks not used very much this year, although most of the students had experienced this material in previous school years.

Mrs. J was conscious of the limitations of the traditional approach as well as the importance of working with materials in some activities. She explained to me that a difficulty she was having referred to the lack of good word problems related to these topics. As she mentioned, "I would love to have problems to work with decimals" as a follow-up to the introduction. She also told me that it would be interesting to have a collection of problems categorized by distinct criteria. "I wish to have them [strategies] organized by skill or topic," to which I replied by telling her about PETTM problem data bank (see the Methods Component Chapter).

Problem-Solving Activities

Problem-solving oriented activities were very distinct from Mrs. J's traditional math periods. These activities usually began with a whole class discussion of a problem. In this phase Mrs. J wanted to be sure that the students understood the problem. Next, the students worked individually. The ones who got stuck were to seek the help of their groupmates. Mrs. J interfered only when no groups came up with a solution. Even in such situations, she would not provide answers. Rather, a whole class discussion of the problem was used to get students to work through the problem to get a solution.

I had the opportunity to observe one problem-solving activity. On that opportunity, the students were working on a problem which was being created for their problem-solving book. (The final form of the problem was: Mrs. Beatie was an animal lover. She had 23 dogs. 17 of the dogs had 4 babies each. 7 dogs came down with yellow spotted fever and died. Mrs. Beatie also had 16 cats. 7 of her cats had 9 babies each. How many animals in all?) The student who first suggested the idea for that problem was in front of the class presenting the problem on a transparency. Many questions arose during her presentation. The problem was seen in different ways, providing for different answers. Answers were checked, the problem was redone, words were eliminated or added to provide clarity and precision. Mrs. J wanted to get it straight, without ambiguities. On the other hand, she did not want to interfere; she wanted the students to decide how the problem should be worded. It was an admirable effort on the part of the students and Mrs. J. In that process, I was also asked to come up with some suggestions. After more discussion the students got the problem into its final form. The teacher told her class that the art teacher would work with them on the illustrations of the problems, causing great excitement in the

class.

In one interview, Mrs. J told me that it was very hard to deal with situations where the students did not seem to be making progress toward consensus on an issue. She said that my observation of the discussion on the dogs and cats problem was the third time there had been debate on that specific problem. Although she was not too concerned with the schedule, she was not convinced that class time was being used productively. It was painful for her to accept that students were moving along without any apparent direction. She did point out that on other occasions the situation would unfold itself naturally through discussion of suggestions presented by the students. In these situations, Mrs. J was happy she had avoided interfering in the discussion process.

Another comment by Mrs. J referred to the students who wrote and presented the problems. It was like teaching according to a whole class participation approach. It was tough for these kids to talk with their classmates while standing in front of the class. On the other hand, these students were getting enormous support from their teacher. And they were proud of their work.

At the time of my observations, problem-solving oriented activities were closely tied to the development of the class problem-solving book. After having worked on multi-step problems, the class was beginning to work on process problems. The decision to spend time developing the problem-solving book contributed to the class "delay" in terms of content coverage. As Mrs. J told me, "being behind made me feel sick. The kids' math [problem-solving] book takes a lot of time but it is worth [it]."

An Integrated Approach

In all my observations I noticed that Mrs. J developed the activities in an integrated way. When mathematical questions arose in class discussions, Mrs. J attempted to build on those situations. I was surprised to hear that she felt that she had not worked on the concept of ratio and proportion. My surprise resulted from my observation of an occasion where the students were checking a homework assignment. The topic was division of decimals by whole numbers and the students were using calculators to check their paper and pencil divisions. When finished, they wanted to figure out what grades they earned. The opportunity to work with proportions was naturally provided. After a student answered that he got A^-, Mrs. J said: "Let's see, 17 in 18 [ratio of correct to total questions], what is this in terms of percent?" "94 percent," answered some students. Mrs. J then asked, "What is this related to if A is between 90 and 100?" The discussion continued and more questions appeared. For example, one student asked "What is .8888 or .89? B^+ or A^-?" This example shows that although Mrs. J did not deal with the concepts of ratio and proportion in the traditional lecture followed by homework sense, the concepts were addressed.

Homework, Evaluation, Group Work and Calculators

Homework in Mrs. J' classroom began, generally, before the students left the school. This provided Mrs. J an opportunity for checking the understanding of the students with respect to the topic taught. Homework was prepared so that the students could get the sense of having been successful by overcoming certain challenges. It was something they could accomplish, yet it was not too easy or boring. Quantity was sacrificed for quality. To Mrs. J, doing fifty long divisions instead of ten was boring for the students who could do it and frustrating for the ones who could not. In the latter case, Mrs. J felt that repetition on something that was not understood would not help to make it understood.

Mrs. J evaluated her students using both objective and subjective measures with the later being at least as important as the former. Mrs. J used objective evaluation measures to hold students accountable. Homework assignments and tests were graded. By subjective evaluation, Mrs. J meant a constant and careful observation of her students' actions and achievement in the classroom. This close observation allowed for the immediate intervention when students were having difficulty and was strongly related to the absence of "surprises" on the tests. Aspects that were taken into consideration during the observations included the amount of time spent on activities, the help obtained from classmates, and the level of frustration of each student. Following one of her most cherished goals, the final objective was to build on success and confidence in the students' mathematical abilities, "a breakthrough to be celebrated."

Group work always took place in the same format in Mrs. J's classroom. The students were arranged in five to six groups of four students each. Besides developing their mathematical abilities, they also had to deal with the social aspects of such interactions. Mrs. J told me that dealing with many problems during the formation and development of the groups allowed her to leave the students working by themselves on many occasions. It seemed to me that the students understood what groups were for. Another aspect mentioned by Mrs. J was that the groups did not turn out to strengthen dependence. Although the students valued the groups, there was also a large amount of independence. Students wanted to show independent methods and solve the problems differently than anybody else.

The use of calculators was a common practice in Mrs. J's classroom although calculator use was usually restricted to the checking of computations done with paper and pencil. When homework assignments involved calculations, the students checked their answers with calculators rather than relying on the teacher for the correct answers. In one of my visits, students checked a homework assignment on division of decimals by whole numbers using this method.

Conclusion

Many of the aspects of mathematics emphasized by PETTM were apparent in Mrs. J's classroom. By getting in touch with new perspectives and being supported by people who believed changes for the better were possible, Mrs. J was able to perceive mathematics education as a multidimensional activity. For instance, her emphasis on developing confidence in the students' mathematical ability provided them the willingness to share with and contribute to the growth of the whole class.

The methodology of group work contributed to the development of an environment where respect and cooperation were present. The students understood that both individual and group work were valid means of interaction; the choice between one or the other was seen as context dependent. Group interaction also provided an opportunity for students to consider the personal characteristics of their group mates. The conversations that I had with some of the students revealed the belief that everybody was able to learn mathematics, that the learning of mathematics differed in manner or pace, and that individual differences were also generated by social prejudices and a lack in basic skills.

The attention placed on word problems provided the students with a broad view of the nature of mathematics. They showed preference for word problems over simple computations. Word problems were considered more interesting because of their connections with real life situations and because information had to be interpreted. Word problems were also perceived as important in that one was able to apply conceptual principles rather than being restricted to the purely mechanical aspects of mathematical operations. Mathematics, however, was still generally interpreted as involving numbers, operations, and special key words. A possible reason for students' emphasis on the importance of key words was that these students had not had much experience in working with process problems. According to Mrs. J, they were just beginning. It would be interesting to observe the evolution of students' conceptualizations of mathematics after they completed the unit on process problems that Mrs. J was planning for them.

The use of manipulatives was appreciated by the students. Although they had started to work with pattern blocks and Cuisenaire rods very recently, it was apparent that the students were trying to relate operations with the materials to the paper and pencil operations they were accustomed to.

At a personal level, Mrs. J was learning how to deal with feelings of uncertainty, a characteristic of situations where change is present. By establishing an environment of mutual support and trust with her students, she was learning to let her fears go away. Previous patterns of control were gradually being substituted by a dynamics of open participation. Mrs. J felt a need to know more about content, materials, and methods. The support that Mrs. J had from PETTM staff made transition to new materials and methods much easier than would otherwise have been the case.

Professionally, Mrs. J was conscious of her role as a teacher. A main issue related to this was the formation of a group of teachers in her school for the purpose of discussing curricular matters. Unwilling to accept what had been taken for granted and feeling the need for consistent support in her practice, Mrs. J considered the formation of such a group as essential. It was only through a mutual support group that Mrs. J felt that innovations in teaching resulting from PETTM would become permanent.

Regarding the formation of a group of teachers in the school, three areas of concern came to mind during our conversations. One referred to the reading and discussing of NCTM's *Curriculum and Evaluation Standards* by the group. Teachers need to reflect and become aware of curriculum objectives, both overt and hidden. The second area was the excessive year-to-year repetition in the textbooks. Mrs. J was concerned about this repetition and mentioned that only through an open and informed discussion could curricular matters be best appreciated, improved, and modified.

The third area of concern was associated with the second one. It referred to the implementation of problem-solving oriented activities in the classroom. As mentioned before, there was normally a slowing down of the "regular" content associated with the utilization of such activities. Mrs. J experienced such a delay and understood that she had to choose between complete coverage of the "standard" topics presented in the textbook and enrichment topics. She was uneasy with the curricular decisions she had to make and wished she had more of an ongoing network of colleagues with whom she could discuss such decisions. The situation becomes more complex if one considers that the ISTEP (Indiana's Statewide Testing Program) is another source of delay in Mrs. J's classroom. As she told me she had to take time to "stop doing what I wanted to do," so that the students could concentrate on the skills asked for in those tests; it was a tense period. Certainly, all three areas deserve the most serious attention.

I finish this report in the hope that it provided a partial view of what was going on in Mrs. J's classroom during my observations and interviews. Despite the brevity of the study, it provided me the opportunity to grasp a combination of the interests and perspectives of the teacher, her students, the traditional curriculum and the ideas presented by PETTM.

Summary and Conclusion

The five teachers interviewed and observed for this part of the project evaluation were not necessarily representative of all teachers who participated in the PETTM project at Highland Park/Dyer Elementary. The five were chosen because they all were involved in the project yet utilized the resources of the project in different ways. The unique nature of the individuals in any large group of teachers makes it impossible to have an inservice program that is ideal for all. If anything, this report shows that dedicated teachers can take the ideas generated through inservice and adapt them to their teaching styles. All five of the teachers described have changed as a result

of PETTM. The change is somewhat different in each, but in all cases it has resulted in improved instructional techniques in mathematics.

Mrs. C, the first grade teacher we observed, was as heavily involved in PETTM as anyone at Highland Park. She attended the inservices and did extensive writing for the *Problem of the Day* book generated as a part of the project. By her own testimony, she now has enough confidence in her mathematics teaching skills to get away from the textbook and do more student oriented activities. In her class students are constructing their own ideas of number both with and without manipulatives. They complete a modest number of activities to practice skills they have learned. Mrs. C's students are continually involved in mathematical *discussion*, through the problem of the day and other activities.

Mrs. C has continual discussions with teachers at Highland Park about teaching mathematics and has served as an inservice instructor and mentor to teachers at other schools. Like all teachers described in this report, Mrs. C felt that having a mathematics resource teacher available in her school was extremely helpful when she was trying out new teaching techniques.

In contrast to Mrs. C, the second grade teacher that was observed (Mrs. L) had only modest involvement with PETTM. Mrs. L attended all of the inservices but was not a part of the summer writing teams. There is no question but that Mrs. L had a difficult group of students to work with and thus discipline took substantial effort on Mrs. L's part. The content she taught was determined by the textbook, although at times she used manipulatives to demonstrate new concepts or skills. She expected students to do seatwork during school and homework in the evening although ambivalence on the part of the students and their parents meant that many finished very few of their assignments. Mrs. L clearly hoped her students would understand the concepts she was presenting but had little faith that they actually did. In conversations with Mrs. L, she commented that she wished she had more time to reflect, both alone and with other teachers, on the ideas presented in the inservices although she did not say when there might have been time available for such reflection. Overall, Mrs. L was supportive of PETTM and its ideals. She achieved some of those ideals although the project did not appear to meet her needs as fully as it did with some of the other teachers.

Mrs. B was the third grade teacher observed as part of the evaluation of the project. Like Mrs. C, she was heavily involved in both the inservice activities and the writing of materials for the project. Our observations and discussions with her indicated that the project had affected her substantially. She noted that as a result of the project she was much more cognizant of what she was doing to help students understand and she was confident that what she was doing was appropriate. We observed that she used a lot of questioning and discussion in her teaching, drawing on students conceptions and misconceptions from manipulatives and from paper and pencil algorithms. She always used the problem of the day to generate discussion and generally

used the textbook only for practice. Mrs. B enjoyed opportunities to discuss and work with other teachers on both teaching techniques and writing problem of the day materials.

The fifth grade teacher observed for the evaluation was Mrs. M. Like Mrs. C and Mrs. B, she had worked two summers writing materials for the project and was providing inservice training and mentoring for teachers in other schools. More so than any of the other teachers interviewed, Mrs. M stated that she was comfortable with her mathematics teaching skills *before* the project began. Several months into the project, however, she began to realize that her drill orientation to teaching was not as effective as she thought it was. By the time of the observations, Mrs. M had gotten away from extensive reliance on drill and taught using questioning and discussion to introduce new ideas. She used manipulatives to introduce some topics and felt she wanted to learn more about how to use manipulatives to introduce and explain other topics. Mrs. M's major interest in mathematics was applying it to real world problems, often using an interdisciplinary approach. The materials she had written for PETTM all involved projects, most of which she had tried in her own class. Like other teachers, Mrs. M wished she had more time to reflect and discuss her involvement in PETTM with other teachers.

Mrs. J, the sixth grade teacher observed for the evaluation was only involved in the project for its first two years. In contrast to the substantial experience of the other teachers observed, Mrs. J had only one year of teaching experience prior to the beginning of the project. Mrs. J saw mathematics as a tool for solving real world problems. The project had little effect on this perception, and Mrs. J became very adept at showing how mathematics could be applied and at using potential applications as motivators for new topics. Mrs. J included a number of estimation and mental math activities in her teaching, and was always using questioning to help students grasp concepts. Mrs. J was confident that she knew what mathematical topics were most important to cover and thus changed the order of topics from those presented in the text. She added her own, often interdisciplinary topics when she felt they were important. One of these topics was the creation of a book of problems by her students. Everyone in the class contributed ideas for problems and drawings to go with the problems they created. The class as a group helped individuals refine their problems into ones that students in other classes could understand. In brief, Mrs. J was very willing to try new methods of teaching. Because she was a relatively new teacher, she was really developing rather than fine tuning her skills as mathematics teacher. The PETTM project seemed to be helpful to her in this regard.

Overall, both observations and discussions with the five teachers observed indicates that (1) they are very competent instructors of mathematics, and (2) the PETTM project contributed substantially to their skills as mathematics teachers. All are different and the way they used the services available from the project were different. Students who have these individuals as teachers will see somewhat different and varied emphases in their instruction. They should, however, get a

well rounded understanding of elementary level mathematical content and its application.

In trying to compare the instructors, several interesting features emerge. First, all have come to the realization that discussion, questioning, and active experimentation are much more effective than the traditional explanation followed by practice form of instruction. This is not to say they do not use traditional explanations or drill. Rather, these are two of a variety of tools they use depending on the topic taught and the level of student interest and prior understanding of that topic.

As these individuals spoke about PETTM, the most recurrent weakness mentioned was the lack of time to reflect, both individually and with other teachers, about the project. PETTM staff were in agreement that more time to discuss ideas would have been ideal. Given the constraints of public education, however, it seemed impossible to find such time. Clearly, those teachers who wrote materials during the summer had an extended period to reflect on mathematics teaching and thus were more affected by the project than other teachers. The materials produced will be useful to many teachers. In addition, however, the process of writing the materials was very influential for the individuals involved.

A third factor that affected the success of the project was the type of students found at Highland Park Elementary. There seemed to be consensus among the teachers studied that problem solving could be taught to motivated, high achieving students. Teaching problem solving to some of the lower achievers at Highland Park met with mixed response. Although discipline was important to Mrs. C and Mrs. B, they seldom mentioned it when talking about how they taught mathematics. Mrs. M and Mrs. J mentioned that discipline was an issue and sometimes kept them from doing some of the activities they wanted to do. Discipline and student motivation, however, did not appear to be major influences in choice of curriculum and activities for Mrs C, Mrs. B, Mrs. M, or Mrs. J. For Mrs. L, discipline and the perception that her students were low to average achievers appeared to be a noticeable impediment to teaching as she wanted. This is a factor that may need to be explored in future projects of this type.

Appendix D: ISTEP Results

Table 1.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in Kindergarten in the Spring of 1988.

1A. Kindergarten (88) vs First Grade (89)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Concepts & Applications	29	51.4	(24.0)	58.6	(26.8)	1.4
Total Reading	30	55.8	(26.2)	43.2	(30.8)	-2.4
Language Expression	30	52.0	(26.5)	49.6	(31.2)	- .4

1B. First Grade (89) vs Second Grade (90)

Variable	*N*	Mean [89]	(SD [89])	Mean [90]	(SD [90])	*t*
Math Computation	37	50.7	(25.0)	38.6	(28.0)	- 2.9*
Math Concepts & Applications	37	62.6	(24.4)	48.1	(26.3)	-3.9*
Total Math	37	56.9	(24.6)	41.5	(27.1)	- 4.1*
Total Reading	37	48.2	(29.1)	43.5	(28.8)	-1.4
Language Expression	36	51.7	(29.4)	48.0	(28.6)	- .9

1C. Kindergarten (88) vs Second Grade (90)

Variable	*N*	Mean [88]	(SD [88])	Mean [90]	(SD [90])	*t*
Math Concepts & Applications	21	48.5	(23.0)	49.9	(25.7)	.3
Total Reading	21	54.1	(29.3)	40.3	(31.0)	-2.3
Language Expression	20	53.0	(28.4)	46.2	(32.2)	-1.3

1D. Second Grade (90) vs Third Grade (91)

Variable	*N*	Mean [90]	(SD [91])	Mean [91]	(SD [91])	*t*
Math Computation	37	38.3	(23.7)	49.6	(28.0)	-2.8*
Math Concepts & Applications	37	53.1	(24.7)	51.5	(29.1)	- .4
Total Math	37	45.4	(24.2)	50.8	(28.0)	1.5
Total Reading	36	45.1	(29.1)	48.7	(25.7)	1.4
Language Expression	36	46.8	(28.1)	45.7	(28.2)	- .4
Total Language	36	45.6	(27.0)	42.5	(25.8)	-1.2
Total Battery	35	46.1	(24.9)	49.0	(24.7)	1.6

*p<.01

Table 2.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in First Grade in the Spring of 1988.

2A. First Grade (88) vs Second Grade (89)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Computation	39	67.1	(21.6)	57.5	(31.9)	-2.2
Math Concepts & Applications	39	71.5	(21.1)	53.8	(29.3)	-4.7*
Total Math	39	71.6	(21.7)	56.8	(30.0)	-4.3*
Total Reading	39	64.9	(22.6)	53.4	(25.4)	-4.0*
Language Expression	39	63.7	(22.8)	50.9	(26.1)	-3.6*

2B. Second Grade (89) vs Third Grade (90)

Variable	*N*	Mean [89]	(SD [89])	Mean [90]	(SD [90])	*t*
Math Computation	31	61.1	(33.2)	47.0	(26.3)	-3.3*
Math Concepts & Applications	31	51.7	(29.0)	57.0	(25.8)	1.5
Total Math	31	58.0	(30.9)	52.8	(27.7)	-1.5
Total Reading	31	56.2	(25.6)	56.4	(24.1)	.1
Total Language	31	59.4	(26.3)	56.3	(27.9)	-1.2
Language Expression	31	55.8	(24.3)	57.5	(28.3)	.5
Total Battery	31	58.2	(28.1)	54.8	(26.0)	-1.7

2C. First Grade (88) vs Third Grade (90)

Variable	*N*	Mean [88]	(SD [88])	Mean [90]	(SD [90])	*t*
Math Computation	31	68.5	(21.4)	49.3	(26.0)	-4.2*
Math Concepts & Applications	31	72.9	(21.2)	59.2	(25.6)	-4.3*
Total Math	31	73.2	(21.4)	55.1	(27.3)	-5.1*
Total Reading	31	67.2	(22.8)	57.3	(24.2)	-2.6
Language Expression	31	65.6	(23.4)	59.0	(27.9)	-1.5

2D. Third Grade (90) vs Fourth Grade (91)

Variable	*N*	Mean [90]	(SD [90])	Mean [91]	(SD [91])	*t*
Math Computation	32	45.3	(23.7)	41.8	(31.4)	-.4
Math Concepts & Applications	32	51.7	(25.3)	46.3	(27.5)	-2.8
Total Math	32	51.7	(25.1)	44.5	(28.9)	-2.1
Total Reading	32	56.6	(23.9)	50.7	(24.4)	-2.3
Total Language	32	53.3	(25.3)	49.9	(25.7)	-1.2
Language Expression	32	53.4	(26.8)	46.5	(26.7)	-2.2
Total Battery	32	53.8	(23.4)	48.4	(26.4)	-2.4

*p<.01

Table 3.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in Second Grade in the Spring of 1988.

3A. Second Grade (88) vs Third Grade (89)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Computation	45	65.6	(28.8)	60.0	(28.5)	-1.2
Math Concepts & Applications	45	60.4	(21.9)	58.5	(25.4)	- .5
Total Math	45	63.8	(24.4)	60.1	(27.6)	-1.0
Total Reading	45	56.3	(27.9)	55.6	(26.2)	- .3
Total Language	45	52.8	(25.0)	52.3	(29.8)	- .2

3B. Third Grade (89) vs Fourth Grade (90)

Variable	*N*	Mean [89]	(SD [89])	Mean [90]	(SD [90])	*t*
Math Computation	30	59.2	(27.5)	52.9	(27.7)	-1.3
Math Concepts & Applications	30	59.0	(22.9)	55.5	(21.0)	-1.1
Total Math	30	59.9	(26.2)	52.5	(23.9)	-2.1
Total Reading	30	56.5	(23.3)	54.8	(25.3)	- .7
Total Language	30	50.8	(28.6)	47.7	(23.7)	-1.1
Language Expression	30	53.6	(26.7)	51.3	(23.7)	- .8
Total Battery	30	55.9	(25.7)	51.6	(24.3)	- 2.0

3C. Second Grade (88) vs Fourth Grade (90)

Variable	*N*	Mean [88]	(SD [88])	Mean [90]	(SD [90])	*t*
Math Computation	29	62.3	(27.6)	54.3	(27.1)	-1.3
Math Concepts & Applications	29	59.1	(17.7)	56.8	(20.1)	- .6
Total Math	29	61.9	(22.1)	53.9	(23.2)	-1.7
Total Reading	29	56.4	(25.6)	55.5	(25.5)	- .3
Total Language	29	52.0	(22.5)	48.4	(23.7)	-1.0

3D. Fourth Grade (90) vs Fifth Grade (91)

Variable	*N*	Mean [90]	(SD [90])	Mean [91]	(SD [91])	*t*
Math Computation	26	50.0	(24.0)	31.3	(23.9)	-4.2*
Math Concepts & Applications	26	51.9	(23.8)	43.9	(29.6)	-2.1
Total Math	26	51.6	(23.0)	37.7	(27.9)	-3.5*
Total Reading	26	58.4	(28.1)	54.4	(27.0)	-1.5
Total Language	26	52.3	(23.2)	44.5	(25.1)	-2.1
Language Expression	26	53.6	(23.1)	40.1	(29.7)	-3.6*

*p<.01

Table 4.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in Third Grade in the Spring of 1988.

4A. Third Grade (88) vs Fourth Grade (89)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Computation	37	49.0	(24.9)	42.5	(22.4)	-1.9
Math Concepts & Applications	37	52.6	(24.5)	50.6	(22.7)	- .5
Total Math	37	50.3	(24.1)	46.5	(23.2)	-1.2
Total Reading	37	54.6	(23.3)	56.1	(21.5)	.5
Total Language	37	56.8	(24.9)	43.3	(22.1)	-5.7*

4B. Fourth Grade (89) vs Fifth Grade (90)

Variable	*N*	Mean [89]	(SD [89])	Mean [90]	(SD [90])	*t*
Math Computation	37	40.6	(21.7)	47.5	(26.7)	2.1
Math Concepts & Applications	37	50.0	(23.9)	47.1	(26.4)	- .8
Total Math	37	45.5	(23.4)	46.7	(26.9)	.4
Total Reading	37	55.2	(22.9)	52.5	(23.4)	- .9
Total Language	37	42.5	(22.8)	46.0	(21.3)	2.0
Language Expression	24	43.3	(25.8)	50.8	(23.5)	2.4
Total Battery	24	47.2	(24.6)	46.7	(24.5)	- .2

4C. Third Grade (88) vs Fifth Grade (90)

Variable	*N*	Mean [88]	(SD [88])	Mean [90]	(SD [90])	*t*
Math Computation	34	47.9	(25.4)	47.1	(26.2)	- .2
Math Concepts & Applications	34	51.5	(24.9)	46.3	(25.7)	-1.2
Total Math	34	49.1	(24.4)	46.0	(26.2)	- .9
Total Reading	34	53.6	(23.4)	51.6	(23.9)	- .8
Total Language	34	55.7	(25.1)	44.8	(20.2)	- 4.7

4D. Fifth Grade (90) vs Sixth Grade (91)

Variable	*N*	Mean [90]	(SD [90])	Mean [91]	(SD [91])	*t*
Math Computation	34	46.1	(24.2)	37.7	(24.1)	-2.0
Math Concepts & Applications	34	48.2	(26.5)	50.1	(24.9)	.5
Total Math	34	47.3	(25.7)	42.3	(23.7)	-1.4
Total Reading	34	52.1	(24.3)	48.1	(20.9)	-1.6
Total Language	34	48.9	(20.6)	44.0	(24.3)	-1.8
Language Expression	34	51.1	(21.9)	44.2	(28.9)	-1.9
Total Battery	34	49.4	(22.7)	44.6	(21.6)	-2.4

*p<.01

Table 5.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in Fourth Grade in the Spring of 1988.

5A. Fourth Grade (88) vs Fifth Grade (89)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Computation	20	48.1	(23.6)	49.7	(28.9)	.3
Math Concepts & Applications	20	43.7	(22.9)	48.9	(24.5)	1.7
Total Math	20	45.6	(23.7)	49.3	(26.4)	1.0
Total Reading	20	47.9	(21.0)	48.1	(20.4)	.1
Total Language	20	37.9	(20.1)	42.9	(24.8)	1.8

5B. Fifth Grade (89) vs Sixth Grade (90)

Variable	*N*	Mean [89]	(SD [89])	Mean [90]	(SD [90])	*t*
Math Computation	15	43.9	(29.0)	35.1	(22.8)	-2.0
Math Concepts & Applications	15	45.1	(23.3)	48.9	(24.0)	1.0
Total Math	15	44.0	(25.5)	41.1	(24.9)	-.8
Total Reading	15	48.2	(18.9)	49.6	(22.0)	.3
Total Language	15	38.0	(24.4)	36.6	(19.9)	-.4

5C. Fourth Grade (88) vs Sixth Grade (90)

Variable	*N*	Mean [88]	(SD [88])	Mean [90]	(SD [90])	*t*
Math Computation	14	48.1	(21.0)	33.6	(22.5)	-2.4
Math Concepts & Applications	14	44.8	(22.3)	49.9	(23.5)	1.6
Total Math	14	46.4	(21.7)	40.5	(24.2)	-1.4
Total Reading	14	50.6	(21.5)	49.5	(21.5)	-.3
Total Language	14	41.1	(22.6)	36.6	(20.1)	-1.1

*p<.01

Table 6.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in Fifth Grade in the Spring of 1988.

6A. Fifth Grade (88) vs Sixth Grade (89)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Computation	30	46.1	(24.7)	51.8	(23.0)	1.4
Math Concepts & Applications	30	50.7	(22.5)	57.2	(18.3)	2.3
Total Math	30	47.7	(22.0)	54.8	(19.1)	2.4
Total Reading	30	61.7	(21.5)	56.7	(21.1)	-2.9*
Total Language	30	55.6	(19.0)	48.2	(19.7)	-3.3*

*p<.01

Table 7.

Highland Park CAT/CTBS Achievement Test Comparisons for Students in First Grade in the Spring of 1990.

7A. First Grade (90) vs Second Grade (91)

Variable	*N*	Mean [88]	(SD [88])	Mean [89]	(SD [89])	*t*
Math Computation	24	50.7	(31.5)	36.3	(27.0)	-2.6
Math Concepts & Applications	24	57.9	(28.5)	58.5	(25.1)	.1
Total Math	24	55.4	(29.6)	46.3	(27.6)	-2.4
Total Reading	24	59.0	(29.7)	56.6	(35.5)	- .6
Language Expression	24	61.6	(31.7)	52.4	(31.6)	-2.2

*p<.01

Appendix E: Problem-solving Test

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Directions for Administering Problem-Solving Instrument

1. There are two parts to the problem-solving instrument. Each part will take 10 to 20 minutes and should be administered on a different day. Part 1 should be given first. Feel free to contact Jackie if you do not feel comfortable giving the test yourself.

2. Tell students that although this is not a "test", they are to do their best work. If you feel that reading will be a problem for some students, please read test items to them.

3. Ask students to *show their work* as much as possible. If they make mistakes, they should simply put an "x" through the mistakes. *Do not erase mistakes.* Circle the final answer for each question.

4. Feel free to answer questions about directions or problem context (e.g. "Do I have to circle both answers?" or "Is tennis a team sport?") Do not answer questions such as "Do I have to add these two numbers." If in doubt about whether to answer a question, use your best judgment.

5. While we do not expect students to "take all day" on these items, please encourage them to take their time and make sure that their answers are correct.

6. On part 2 of the test, the directions for the first problem read:

> You don't need to solve this problem. Just circle the numbers you need to solve the problem. (You may solve the problem if you wish.)

The direction to "solve the problem if you wish" is intended as an option for students who used to doing problems but not used to specifically stating which information in a problem is unnecessary.

7. **When papers are collected, please make sure students' *first* and *last* names are legible. If students have not circled an answer, please ask them to do so** (unless, of course, the student does not know enough to attempt the problem.)

First Grade - Part 1

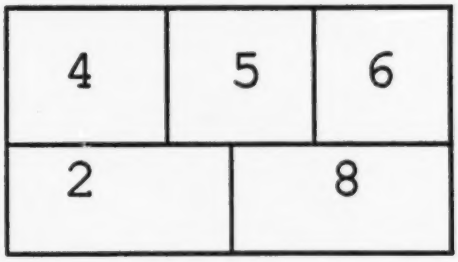
Name _____ Teacher _____

SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

1. There are 10 cars at school. 4 cars drive away. How many cars are still at school?

2. Steve counted 25 cans of pop and 15 boxes of cookies. Then he counted 12 more cans of pop. How many cans of pop did Steve count?

3. Linda threw 2 bean bags in the boxes. She got 7 points. Write the numbers where her bean bags could have landed.



First Grade - Part 2

Name _____ Teacher _____

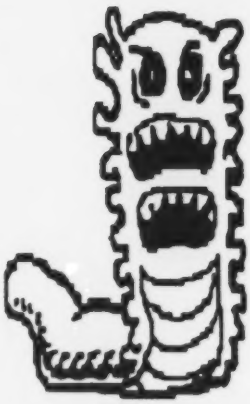
1. YOU DON'T NEED TO SOLVE THIS PROBLEM. JUST CIRCLE THE NUMBERS YOU NEED. (You may solve the problem if you wish.)

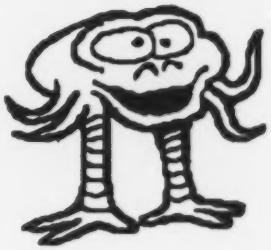
   Sam read 6 books.
   Jo read 5 books.
   Jason read 8 books.
   How many books did Sam and Jason read?

2. SOLVE THIS PROBLEM. SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.
   John bought 10 pencils.
   The pencils were red and blue.
   One way to buy 10 pencils is to get 4 red pencils and 6 blue pencils.
   List **all the other ways** John could buy 10 pencils.

3. SOLVE THIS PROBLEM.

Can you help me find my monster?
My monster has 2 feet.
My monster has no arms.
My monster has no nose.
Draw a circle around my monster.





Second Grade - Part 1

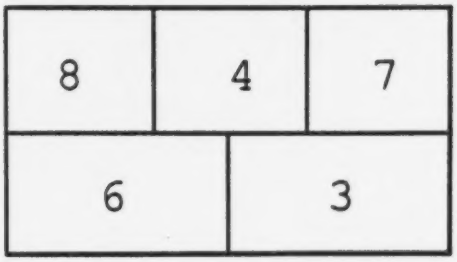
Name _____ Teacher _____

SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

1. Kim has 8 crayons. She has 6 pencils. How many more crayons than pencils does she have?

2. Bobby counted 46 red birds and 12 rose bushes in the park. He counted 18 blue birds. How many birds did Bobby count?

3. Linda threw 3 bean bags and each landed in a box. She got 13 points. Write the numbers where her bean bags could have landed.



Second Grade - Part 2

Name _____ Teacher _____

1. YOU DON'T NEED TO SOLVE THIS PROBLEM. JUST CIRCLE THE NUMBERS YOU WOULD NEED. (You may solve the problem if you wish.)

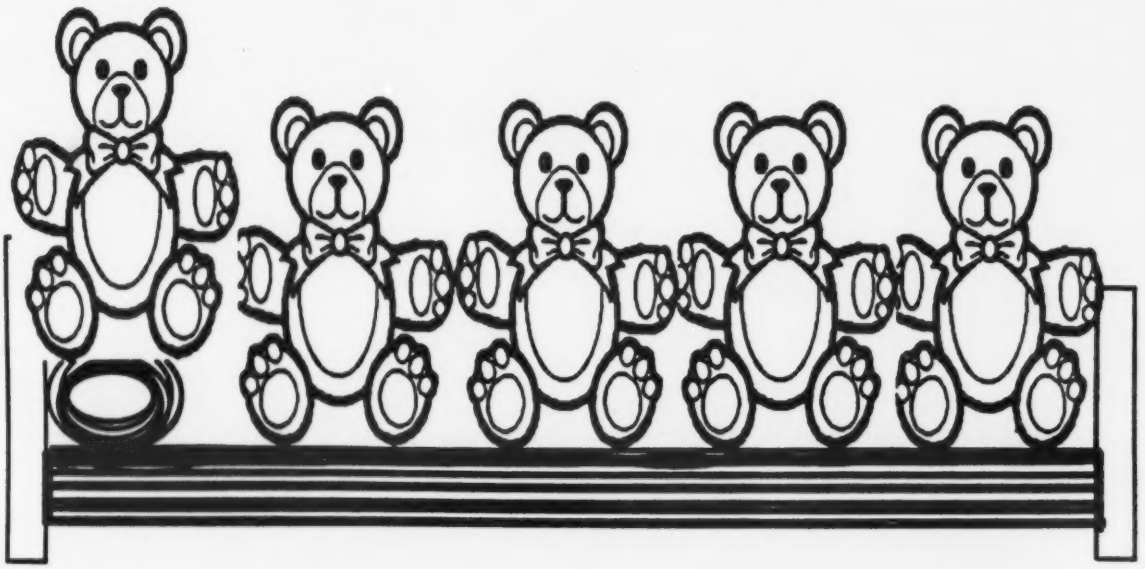
   Frank has 23¢, Kelly has 14¢, and Beth has 37¢. How much money do Frank and Beth have together?

2. SOLVE THIS PROBLEM. SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

   A squirrel worked for 5 days gathering acorns. On the first day, she gathered 6. Each day the squirrel gathered 3 more acorns than the day before. How many acorns did the squirrel find on the 4th day?

3. SOLVE THIS PROBLEM.

Sonya has 5 Teddy bears. One day she lined them up on her bed. Jacob is sitting on the pillow. Toby is at the end of the bed. Jim is between Leo and Fred. Fred is next to Jacob. Find each Teddy bear. Write his name on the line below him.



Third Grade - Part 1

Name _____ Teacher _____

SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

1. Martha planted 334 seeds. Only 216 grew. How many seeds did not grow?

2. Farmer Brown has 4 blue bags of apples and 3 yellow bags of apples. There are 21 apples in each blue bag and 23 apples in each yellow bag. How many apples are there altogether in the blue bags?

3. Wilbur asked his mom to tell him where the oatmeal cookie recipe was in the cookbook. She said, "The recipe is on 2 pages next to each other. When you add the 2 page numbers, you get 35." On what pages will Wilbur find the cookie recipe?

Third Grade - Part 2

Name _____ Teacher _____

1. YOU DON'T NEED TO SOLVE THIS PROBLEM. JUST UNDERLINE THE NUMBERS YOU WOULD USE TO SOLVE IT. (You may solve the problem if you wish.)

   Ed weighs 74 pounds. His mother weighs 125 pounds. His little sister weighs 23 pounds. How much do Ed and his sister weigh together?

2. SOLVE THIS PROBLEM. SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

   A bus made 4 stops on its way to school. At the first stop it picked up 2 kids. At each of the next stops it picked up 3 more kids than the stop before. How many kids did the bus pick up on its way to school?

3. SOLVE THIS PROBLEM.

Tomas, Katie, Nick, Ronnie, and Dusty each have a different favorite animal.

1. Tomas' and Katie's favorite animals live in the jungle.
2. Katie's favorite animal loves to swing through the trees.
3. Nick likes to ride his favorite animal.
4. Ronnie's favorite animal hops.

Write each child's name under his or her favorite animal.

Fourth Grade - Part 1

Name _____ Teacher _____

SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

1. There are 50 newspapers to deliver. 5 newspaper carriers will each deliver the same number. How many will each person deliver?

2. Trash cans cost $10 each. Stands for the cans cost $40 each. Picnic tables cost $53 each. A playground will have 5 trash cans and 5 stands, and 3 picnic tables. For the whole playground, how much more will the stands cost than the trash cans?

3. Wilbur asked his mom to tell him where the oatmeal cookie recipe was in the cookbook. She said, "The recipe is on two pages next to each other. When you add the two page numbers, the sum is 113." On what pages will Wilbur find the cookie recipe?

Fourth Grade - Part 2

Name _____ Teacher _____

1. YOU DON'T NEED TO SOLVE THIS PROBLEM. JUST CIRCLE THE NUMBERS YOU WOULD USE TO SOLVE IT. (You may solve the problem if you wish.)

   Monica played 6 games and scored a total of 67 points. Bridget played 5 games and scored a total of 45 points. How many more points did Monica score than Bridget?

2. SOLVE THIS PROBLEM. SHOW YOUR WORK AND CIRCLE YOUR ANSWER.

   Every ice cream sundae at the Dairy Delite has one flavor of ice cream and one flavor of topping. The choices for ice cream are chocolate, vanilla, and strawberry. The choices for topping are fudge or butterscotch. For example, you can order a fudge sundae with vanilla ice cream. How many different sundaes can be ordered?

3. SOLVE THIS PROBLEM.

Billy, Ivan, Tomas, Zeke, and Tony each have a different favorite sport.
Billy and Ivan play on teams.
Ivan and Tomas use round balls in their games.
Zeke's sport is done near water.

Write each boy's name by his sport.

Fifth Grade - Part 1

Name _____ Teacher _____

SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER

1. There are 672 newspapers to deliver. 8 newspaper carriers will each deliver the same number. How many will each person deliver?

2. In 1895, Thomas Conneff set a world record by running a mile in 4 min 15.6 sec. Jim Ryan was the first American who ran a mile in less than 4 minutes. In 1966, he ran a mile in 3 min 51.3 sec. How many years separated these two important records for U.S. runners?

3. Last night I was watching a PeeWee football game near my house. I saw some boys and dogs run past. There were 12 boys and dogs altogether. I counted their legs and got 40. How many boys and how many dogs ran past me?

Fifth Grade - Part 2

Name _____ Teacher _____

1. DON'T SOLVE THIS PROBLEM. JUST CIRCLE THE NUMBERS YOU WOULD USE TO SOLVE IT. (You may solve the problem if you wish.)

   Luana and Diana had 324 buttons in their button collection and 214 shells in their shell collection. They decided to split their shell collection into 2 parts so that each person could keep the same number of shells in her own room. How many shells did Diana get?

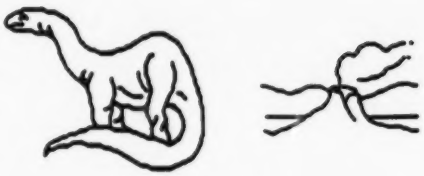
2. SOLVE THIS PROBLEM. SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

   The Tastee-Treat serves ice cream in four different flavors: vanilla, chocolate, strawberry, and coffee. A double-dip cone has two different flavors of ice cream (for example, vanilla and chocolate). How many different double -dip cones are possible?

3. SOLVE THIS PROBLEM.

Use the clues to help you discover the name of each dinosaur. Write the name of each dinosaur on the line underneath it.

1. One of the dinosaurs is named Diana.
2. Donnie is between the trees.
3. David is closest to the volcano.
4. Denise is not next to David.
5. Delsie is between David and Donnie.



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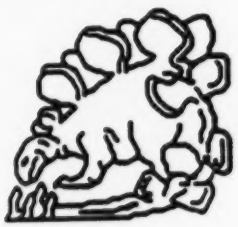

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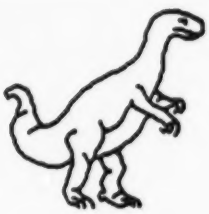

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Sixth Grade - Part 1

Name _____ Teacher _____

SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

1. The sixth-grade class gave a party. Altogether the party food cost $23.03. If seven students who planned the party shared the food expenses equally, how much did each student pay?

2. Sharon worked 215 hours in April, 205 hours in May, and 180 hours in May. She spends half of her time typing and one-fourth of her time in meetings. How many hours did she spend typing in April and May?

3. Jennifer's dad decided to go on a diet. Jennifer's mom made a deal with Jennifer's dad. She said, "I'll put $10 in the cookie jar for every day you lose weight, but I'll take out $5 for every day you don't lose weight." At the end of 10 days, there was $55 in the cookie jar. How many days did Jennifer's dad lose weight?

Sixth Grade - Part 2

Name _____ Teacher _____

1. YOU DON'T NEED TO SOLVE THIS PROBLEM. JUST CIRCLE THE NUMBERS YOU WOULD USE TO SOLVE IT. (You may solve the problem if you wish.)

   Magellan sailed from Spain to Guam in about 125 days. In 1620, the Mayflower crossed the Atlantic Ocean in about 25 days. Today an ocean liner crosses the Atlantic in about 5 days. About how many times faster does an ocean liner today cross the Atlantic Ocean than the Mayflower could cross it?

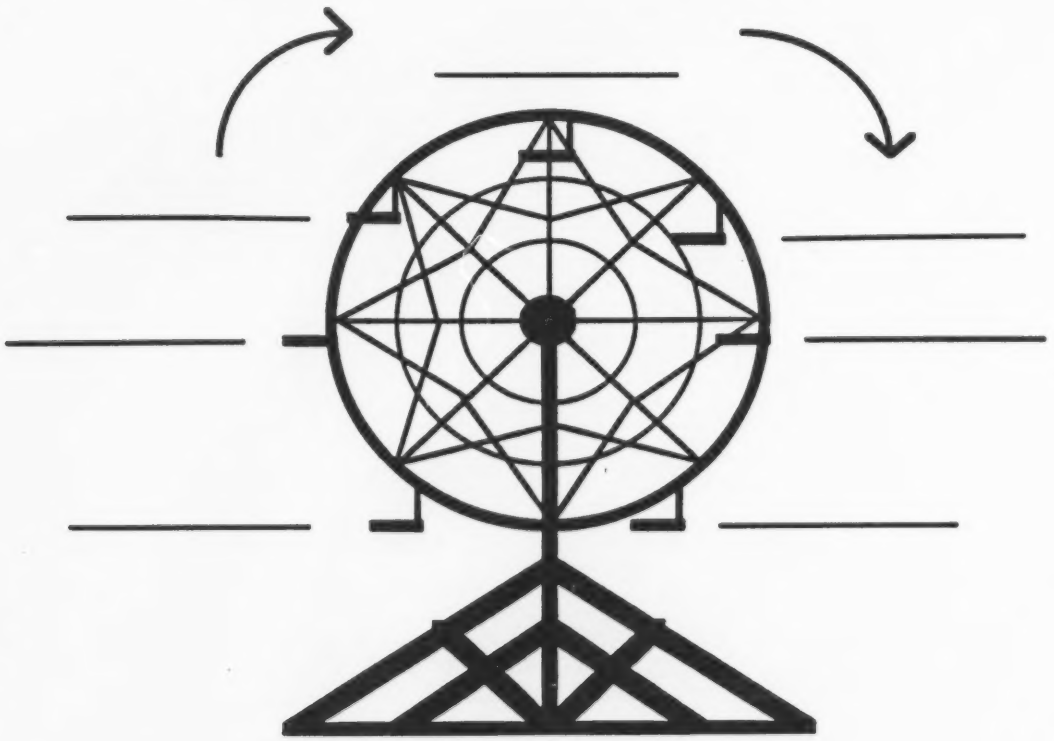
2. SOLVE THE PROBLEM. SHOW YOUR WORK AND CIRCLE YOUR FINAL ANSWER.

   Andrea wants to buy a snack in a snack machine that will take only quarters, dimes, and nickels. If each snack costs 35¢ and she must use the exact change, how many different combinations of coins can Andrea use to buy a snack?

3. SOLVE THE PROBLEM.

Seven friends are riding the Ferris wheel at the county fair. Read the clues and then write the riders' names on the lines next to the cars in which they are riding.

1. **The Ferris wheel is moving clockwise (in the direction of the arrows).**
2. **Jon is in the lowest car moving upward.**
3. **Jennifer, in the highest car, has the best view of the city.**
4. **Monica is between Pete and Maggie.**
5. **Carol is between two boys. Frank is one of them.**
6. **Jennifer is next to Pete.**



Scoring System for Highland Park Student Problem Solving Test

General Guidelines:

If an answer is hard to read (illegible), give the student benefit of the doubt if one interpretation of the written answer is the correct one. When evidence is needed of how an answer was arrived at (for partial credit), reasonably clear documentation that the student understood what to do is needed. If work has been erased, it can be used for partial credit if it is still readable.

Grade 1, Part 1:

#1 2 = correct answer (6) or representation of six
1 = incorrect answer but correct operation shown or apparent (10-4) or counting down, etc.)
0 = any other incorrect answer

#2 2 = correct answer (37)
1 = incorrect answer but correct numbers and operation used or apparent (25 + 12 or counting on, etc.)
0 = any other incorrect answer

#3 1 = correct answer (2 and 5)
0 = any other answer

Grade 1, Part 2:

#1 1 = circle or identify 6 and 8 *or* correctly solve the problem (14)
0 = any other answer

#2 3 = all eight correct combinations or at least 6 correct combinations and a logical progression shown [note: (10,0) and (0,10) are not wrong, but are not counted in the list of correct combinations]
2 = four to five correct combinations
1 = two to three correct combinations
0 = less than two correct combinations

#3 1 = correct answer (circled figure in upper left corner)
0 = any other answer

Grade 2, Part 1:

#1 2 = correct answer (2)
1 = incorrect answer but correct operation shown or apparent (8 - 6 or counting down, etc.)
0 = any other incorrect answer

#2 2 = correct answer (64)
1 = incorrect answer but correct numbers and operation used or apparent (46 + 18 or counting on, etc.)
0 = any other incorrect answer

#3 1 = correct answer (7, 3, 3) or (3, 4, 6)
0 = any other answer

Grade 2, Part 2:

#1 1 = circle or identify 23 and 37 *or* correctly solve the problem (60)
0 = any other answer

#2 3 = correct answer (15)
2 = 12 or 18 (miscounted days) or 2 terms of the sequence written out (6,9 or 6,9,12)
1 = 3 or 42 (misinterpreted the question - 42 = 6+9+12+15)
0 = any other incorrect answer

#3 2 = correct answer (Jacob, Fred, Jim, Leo, Toby) Note: spelling is not important as long as you can tell which bear is being referred to.
1 = Jacob, Fred, Leo, Jim, Toby *or* Jacob, Leo, Jim, Fred, Toby *or* Toby, Leo, Jim, Fred, Jacob *or* having 3 or 4 names correctly filled in where filling in the others would **have to result** in a partially or fully correct answer. Ex: Jacob ___ Jim ____ Toby is worth 1 point because the only options give the correct or one of the 1 point incorrect answers. If a student writes a name that does not appear in the problem or writes multiple names in any given answer, treat each of these responses as though it were blank.
0 = any other incorrect answer

Grade 3, Part 1:

#1 2 = correct answer (118)
1 = incorrect answer but correct operation shown or apparent (334-216)
0 = any other incorrect answer

#2 2 = correct answer (84)
1 = incorrect answer but correct numbers and operation used or apparent (21 + 21 + 21 + 21 or 4 X 21)
0 = any other incorrect answer

#3 3 = correct answer (17 and 18) Note: Some students may write the two numbers in very close proximity. If it looks like 1718, give them full credit.
2 = 16 and 17 *or* 18 and 19 (1617 or 1819 is also worth 2 points)
1 = satisfied one condition of the problem (answers sum to 35 or are consecutive integers)
0 = any other incorrect answer

Grade 3, Part 2:

#1 1 = circle or identify 74 and 23 *or* correctly solve the problem (97)
0 = any other answer

#2 2 = correct answer (26)
1 = evidence of understanding the pattern (2 + 5 + 8 + 11)
0 = any other incorrect answer

#3 2 = correct answer (dog-Dusty; rabbit-Ronnie; monkey-Katie; lion-Tomas; horse-Nick)
Note: Spelling is not important as long as you can tell which animal is being referred to.
1 = Katie and Tomas positions switched, all others correct *or* having 3 or 4 names correctly filled in where filling in the others would have to result in a partially or fully correct answer. Ex: Dusty Ronnie _____ _____ Nick is worth 1 point because the only options give the correct or one of the 1 point incorrect answers. If a student writes a name that does not appear in the problem or writes multiple names in any given answer, treat each of these responses as though it were blank..
0 = any other incorrect answer

Grade 4, Part 1:

#1 2 = correct answer (10)
1 = incorrect answer but correct operation shown or apparent (50+5)
0 = any other incorrect answer

#2 3 = correct answer ($150)
2 = incorrect answer but correct numbers and operations used or apparent (40 x 5 - 10 x 5 or [40 - 10] x 5) Note: This category includes incorrect decimal placement - $1.50 or $15.0.
1 = one error in understanding ($30 or $250)
0 = any other incorrect answer

#3 3 = correct answer (56 and 57)
2 = 55 and 56 or 57 and 58
1 = satisfied one condition of the problem (answers sum to 113 or are consecutive integers)
0 = any other incorrect answer

Grade 4, Part 2:

#1 1 = circle or identify 67 and 45 or correctly solve the problem (22)
0 = any other answer

#2 3 = correct answer (6)
2 = 12 (answer obtained by doubling possibilities) or 5 (missing one possibility) Note: these answers must show evidence of an organized list. Merely adding 3 + 2 does not earn credit. Also, an answer of 5 that gives no indication of how the student arrived at the answer is not given credit.
1 = 3 or 4 (missing two or three possibilities) or an organized list with too many possibilities
0 = any other incorrect answer

#3 3 = correct answer (weights-Tony; fish-Zeke; basketball-Ivan; football-Billy; tennis-Tomas) Note: Spelling is not important as long as you can tell which sport is being referred to.
2 = Tony, Zeke, Tomas, Billy, Ivan (interchanging Tomas and Ivan on the assumption that tennis is a team sport)
1 = Tony, Zeke, Billy, Ivan, Tomas *or* Tony, Zeke, Tomas, Ivan, Billy *or* having 3 or 4 names correctly filled in where filling in the others would result in a partially or fully correct answer. Ex: Tony, Zeke ___ Billy ___ is worth 1 point because the only options give the correct or one of the1 or 2 point incorrect answers. If a student writes a name that does not appear in the problem or writes multiple names in any given answer, treat each of these responses as though it were blank.
0 = any other incorrect answer

Note: We thought the directions for #3 should be changed to stress that different names should be used for each blank, but it is not clear that the directions actually were changed.

Grade 5, Part 1:

#1 2 = correct answer (84)
1 = incorrect answer but correct operation shown or apparent (672+8)
0 = any other incorrect answer

#2 2 = correct answer (71)
1 = incorrect answer but correct numbers and operation used or apparent (1966 - 1895)
0 = any other incorrect answer

#3 3 = correct answer (4 boys and 8 dogs or 4 and 8)
2 = reversing boys and dogs (8b,4d) or off by one (3b,9d or 5b,7d)
Note: labels of boys and dogs or b and d required for two points but not for full credit or for one point. Lack of labels on anything except the correct answer (4 and 8) is worth one point (3,9 or 5,7 or 39 or 57 or 48 or 84 is worth one point)
1 = satisfy one condition of the problem (answers sum to 12 or obvious that legs sum to 40) or lack of labels on anything except correct answer (see above)
0 = any other incorrect answer

Grade 5, Part 2:

#1 1 = circle or identify 214 and 2 *or* correctly solve the problem (107)
0 = any other answer

#2 3 = correct answer (6 or 12)
2 = 11 or 5 (missing one possibility)
1 = 10 or 16 (counts 2 dips of the same flavor) or 4 or 10 (missing two possibilities) or an organized list with too many possibilities
0 = any other incorrect answer

#3 2 = correct answer

Diana David

Delsie

Denise

Donnie

Note: Spelling is not important as long as you can tell which dinosaur is being referred to.

1 =

David

Delsie

Diana

Denise

Donnie

or

David

Denise

Delsie

Diana

Donnie

or having 3 or 4 names correctly filled in where filling in the others would result in a partially or fully correct answer. Ex:

David

_____ _____

Denise

Donnie

is worth 1 point because the only options give the correct or one of the 1 point incorrect answers. If a student writes a name that does not appear in the problem or writes multiple names in any given answer, treat each of these responses as though it were blank.

0 = any other incorrect answer

Grade 6, Part1:

#1 2 = correct answer (3.29 or $3.29)
1 = incorrect answer but correct operation shown or apparent (23.03+7 or answer with incorrectly placed decimal)
0 = any other incorrect answer

#2 3 = correct answer (210)
2 = incorrect answer but correct operations shown or apparent ([215+205]+2 or [215+2]+[205+2]
1 = one incorrect piece of information used) acceptable responses are 105, 192 1/2, 197 1/2, 300)
0 = any other incorrect answer

#3 3 = correct answer (7)
2 = confusing the days he lost weight with those he didn't (3 or 3 and 7)
1 = appropriate strategy used or apparent
0 = any other incorrect answer

Grade 6, Part 2:

#1 1 = circle or identify 5 and 25 *or* correctly solve the problem (5)
0 = any other answer

#2 3 = correct answer (6) **Note:** If a correct list is provided but the number 6 is not written, give full credit. It the list does not agree with the answer given, grade according to answer rather than the list.
2 = 5 or a list of 5 possibilities
1 = 4 or a list of 4 possibilities
0 = any other incorrect answer

#3 2 = correct answer (clockwise from top: Jennifer, Pete, Monica, Maggie, Jon, Carol, Frank)
Note: Spelling is not important as long as you can tell which person is being referred to.
1 = (clockwise from top:)
Jennifer, Pete, Monica, Maggie, Jon, Frank, Carol
or
Jennifer, Pete, Maggie, Monica, Jon, Carol, Frank
or
Jennifer, Pete, Carol, Frank, Jon, Monica, Maggie
or
Jennifer, Pete, Carol, Frank, Jon, Maggie, Monica
or
Jennifer, Maggie, Monica, Pete, Jon, Carol, Frank
or
Jennifer, Frank, Carol, Jon, Maggie, Monica, Pete
or
having 5 or 6 names correctly filled in where filling in the others would result in a partially or fully correct answer. Ex:
Jennifer, Pete, ____, ____, Jon, Carol, Frank is worth 1 point because the only options give the correct or one of the 1 point incorrect answers. If a student writes a name that does not appear in the problem or writes multiple names in any given answer, treat each of these responses as though it were blank.
0 = any other incorrect answer

Table 1.

Highland Park Problem-Solving Test Data for First Grade Students[a]

	1989			1990			1991		
Item Number	*N*	Mean	SD	*N*	Mean	SD	*N*	Mean	SD
Part 1, #1	49	1.84	.55	39	1.92	.35	32	1.91	.39
Part 1, #2	49	.73	.90	39	.85	.99	32	1.31	.93
Part 1, #3[b]	49	1.56	.80	39	.54	.51	32	.59	.50
Part 2, #1	49	.54	.50	37	.62	.49	35	.74	.44
Part 2, #2	49	1.16	.90	37	1.46	1.02	35	1.17	.89
Part 2, #3[c]				37	.97	.16	35	1.00	.00

[a]Item numbers correspond to problems listed earlier in this Appendix. Because of minor changes in wording and scoring of the problems from year to year, and because the population changed from year to year, comparing statistics across years is not appropriate.

[b]This item had a maximum score of 2 points in 1989 but a maximum score of 1 point in 1990 and 1991.

[c]Part 2, #3 was not administered in 1989.

Table 2.

Highland Park Problem-Solving Test Data for Second Grade Students[a]

	1989			1990			1991		
Item Number	*N*	Mean	SD	*N*	Mean	SD	*N*	Mean	SD
Part 1, #1	29	1.34	.94	42	1.81	.59	27	1.48	.89
Part 1, #2	29	.48	.87	42	1.24	.96	27	1.22	.97
Part 1, #3[b]	29	1.28	.92	42	.81	.40	27	.56	.51
Part 2, #1	29	.43	.50	42	.88	.33	27	.78	.42
Part 2, #2	29	.52	1.02	42	2.36	1.10	27	1.07	1.30
Part 2, #3[c]				42	1.79	.52	27	1.56	.75

[a]Item numbers correspond to problems listed earlier in this Appendix. Because of minor changes in wording and scoring of the problems from year to year, and because the population changed from year to year, comparing statistics across years is not appropriate.

[b]This item had a maximum score of 2 points in 1989 but a maximum score of 1 point in 1990 and 1991.

[c]Part 2, #3 was not administered in 1989.

Table 3.

Highland Park Problem-Solving Test Data for Third Grade Students[a]

	1989			1990			1991		
Item Number	*N*	Mean	SD	*N*	Mean	SD	*N*	Mean	SD
Part 1, #1	38	1.75	.61	37	1.51	.77	53	1.53	.72
Part 1, #2	38	1.11	1.01	37	.92	1.01	53	.87	.98
Part 1, #3	38	.86	1.31	37	.78	1.18	53	.96	1.21
Part 2, #1	38	.61	.48	37	.81	.40	52	.79	.41
Part 2, #2	38	.22	.62	37	.05	.33	52	.23	.58
Part 2, #3[b]				37	1.57	.65	52	1.56	.57

[a]Item numbers correspond to problems listed earlier in this Appendix. Because of minor changes in wording and scoring of the problems from year to year, and because the population changed from year to year, comparing statistics across years is not appropriate.

[b]Part 2, #3 was not administered in 1989.

Table 4.

Highland Park Problem-Solving Test Data for Fourth Grade Students[a]

	1989			1990			1991		
Item Number	*N*	Mean	SD	*N*	Mean	SD	*N*	Mean	SD
Part 1, #1	46	1.61	.80	33	1.24	.94	47	1.30	.93
Part 1, #2	46	.86	1.15	33	1.36	1.37	47	.57	1.12
Part 1, #3	46	1.11	1.30	33	1.79	1.32	47	1.36	1.26
Part 2, #1	46	.73	.44	33	.73	.45	45	.69	.47
Part 2, #2	46	1.91	1.21	33	2.03	1.24	45	1.73	1.37
Part 2, #3[b]				33	1.42	1.44	45	1.22	1.33

[a]Item numbers correspond to problems listed earlier in this Appendix. Because of minor changes in wording and scoring of the problems from year to year, and because the population changed from year to year, comparing statistics across years is not appropriate.

[b]Part 2, #3 was not administered in 1989.

Table 5.

Highland Park Problem-Solving Test Data for Fifth Grade Students[a]

	1989			1990			1991		
Item Number	*N*	Mean	SD	*N*	Mean	SD	*N*	Mean	SD
Part 1, #1	8	2.00	.00	36	1.64	.72	26	1.46	.76
Part 1, #2	8	1.38	.92	36	1.58	.81	26	1.92	.39
Part 1, #3	8	1.50	1.60	36	1.75	1.25	26	2.04	1.31
Part 2, #1	8	3.00	.00	39	.72	.46	29	.66	.48
Part 2, #2	8	1.00	.00	39	2.15	1.29	29	2.21	1.15
Part 2, #3[b]				39	1.77	.43	29	1.66	.48

[a]Item numbers correspond to problems listed earlier in this Appendix. Because of minor changes in wording and scoring of the problems from year to year, and because the population changed from year to year, comparing statistics across years is not appropriate.

[b]Part 2, #3 was not administered in 1989.

Table 6.

Highland Park Problem-Solving Test Data for Sixth Grade Students[a]

	1989			1990			1991		
Item Number	*N*	Mean	SD	*N*	Mean	SD	*N*	Mean	SD
Part 1, #1	36	1.79	.58	17	1.76	.56	32	1.53	.67
Part 1, #2	36	1.76	1.38	17	1.47	1.50	32	1.19	1.42
Part 1, #3	36	.83	1.36	17	.47	.94	32	.78	1.31
Part 2, #1	36	1.90	1.18	16	.81	.40	35	.46	.51
Part 2, #2	36	.47	.51	16	1.87	1.31	35	1.57	1.29
Part 2, #3[b]				16	1.63	.81	20	1.75	.64

[a]Item numbers correspond to problems listed earlier in this Appendix. Because of minor changes in wording and scoring of the problems from year to year, and because the population changed from year to year, comparing statistics across years is not appropriate.

[b]Part 2, #3 was not administered in 1989.

Appendix F: Student Belief Interview Schedules

Spring 1990 Dyer Student Interview Schedule
Grade 2 (or late grade 1)

Interviewer _____ **Student** _____

Time and Date _____

Reminders for interviewers:
Schedule 30 minutes for each interview and allow 15 minutes between interviews to complete your notes. Whenever you prompt a student to get a response, put a "P" before writing that response. In many cases, it will not be possible to write down exactly what a student says. However, whenever you have enough time to include verbatim comments, please do so (put exact wording in quotes). Notes about student non-verbal responses are also appropriate (e.g., nod of the head can be interpreted as a yes or no, confused look should be noted to indicate student didn't understand question, etc.)

Interviewer introduces him/herself and says: *I am interested in your opinions and feelings about some of the kinds of school work you do. The things we talk about will help me understand better how you feel about school work. You can tell your parents and teachers about the things we discuss if you want to but I won't be telling them anything that you say. I hope you will be at ease and give me honest responses to the questions I ask. If there are any questions that you don't want to answer, just say you don't want to answer them. There are no right or wrong answers for my questions, I simply want your feelings. OK?* Allow child to question you about the interview. If necessary, make additional "small talk" to put child at ease.

1. Background information/mindset of the student

Do you like school? Why or why not?

What is your favorite subject in school? Why?

Are there any subjects you don't like? Why?

If math is not mentioned, ask *Do you enjoy math?* Probe to get an idea of student's general feelings toward math.

How often do you talk with your parents about school? Tell me about the last time you talked with your parents about school.

Do your parents expect you to do well in school? Do they expect you to do well in math?

At home, do you ever watch "Sesame Street" or "Square One," play counting games, card games, talk about numbers, or do other math activities?

2. Usefulness/importance of mathematics

Do you think it is important to learn mathematics? Why? (Probe for both "in school" uses such as "If don't learn it I'll have to go to summer school" and "real world" uses such as "I need to know if I have enough money when I go to the grocery store.")

Do you know anyone who uses math? (If yes) Who? How do they use math?

3. Self confidence

Do you do well in math? Explain.

4. Solving computational and word problems

I would like you to do several math problems for me. First, I would like you to read each problem out loud. I'll help you with the reading if needed. Before you start each problem, I would like you to tell me if you think you can do it. Then, as you work on each problem, I would like you to explain to me exactly what you are thinking and why you are doing everything you do. I want to know everything you think about as you do each problem. When you have finished each problem, I am going to ask you how you felt about doing that problem. OK?

Let student see first problem and help with reading if necessary. Ask the student if he (she) thinks he (she) can do it and why. Then allow student to solve the problem. Prompt student to "think aloud" to see if he (she) relies on the key words and/or any other "cookbook" type of step-by-step solution to the problem. Ask if the student has done similar problems in class. Look for examples of metacognitive monitoring. Be friendly and give hints only when absolutely necessary. (Make sure any hints and/or prompts are noted below.) When student will continue no farther, ask him (her) if he (she) thinks the answer is correct and why. At this point, show the "continuum" of smile/frown faces and ask the student to select the face that expresses his (her) feelings about the problem. Ask why that face was selected. If the student asks, you may finish discussion of the problem by telling the student whether his (her) answer is correct

Continue with other problems, recording the students' comments and your observations of what the student is doing to solve each problem.

Note: Problems will be on separate sheets of paper but interviewer notes should be recorded here.

1.
```
  2
  4
 +6
```

big frown little frown no expression little smile big smile

2. Mary has 14 gum drops. She gets 5 more gum drops. How many gum drops does she have in all?

big frown little frown no expression little smile big smile

3.
```
 75
+12
```

big frown little frown no expression little smile big smile

4. 10 children were at the park but three went home. 5 of the children were boys and 5 were girls. How many children were still at the park?

big frown little frown no expression little smile big smile

5. Ed was baking cookies. His cookies were shaped like circles, stars, and hearts. He made 1 more circle cookie than star cookie and 1 more heart cookie than circle cookie. If he made 5 circle cookies, how many cookies did he make in all?

big frown little frown no expression little smile big smile

5. Importance of word problems

Do you solve word problems in class?

Why do you think your teacher has you solve word problems? (Probe to see if student believes word problems help you to understand math better, if they help you think logically, if they are good for applying math outside of school, etc.)

6. Task involvement vs. ego involvement ("math mind"/importance of effort)

The point of the following questions is to get at students' opinions on the extent to which anyone can learn mathematics and about whether they judge themselves in relation to other students. Thus, in addition to answers to the questions, note comments about how they know who the best students are (and why) and whether one can be good at mathematics without having to try very hard.

Are there any students who just aren't smart enough to be good at math or can every student learn math if they try hard enough? Don't ask for or accept specific names.

Why do you think there are some students who can't do very well in math? **OR** *How do you know everyone can learn math?*

7. Conceptual understanding/number sense

Tell me what you do during math class.
Probe for comments about manipulatives, amount of time teacher spends explaining new lessons, amount of time spent doing or grading seatwork, etc.

What kinds of things does your teacher do to help you when you're having trouble in math? Look for examples of manipulative or pictorial representations of number and place value, "real life" examples, use of "sample" problems, etc.

If student does not mention manipulatives, point to pattern blocks. *Have you ever seen these before?*

If yes, *Do you use these in class? Explain.*

Do you ever get an answer to a math problem but not really understand if you have done the problem correctly? Probe for additional details, particularly what it means to the child to "understand" mathematics. *How does not understanding a problem make you feel?*

I am going to show you some math materials and pictures. I want you to explain to me what they are and what they mean. (The purpose of these questions is to determine, as much as possible, whether the student can relate abstract number to some concrete or pictorial image and whether the student believes that such representations are useful.)

Show a base-10 block diagram of 67 from the second-grade Harcourt Brace Jovanovich (1985) text. *Do you know what number this picture represents?*

Get out base-10 blocks. *Have you used these blocks before? Can you explain to me how these blocks relate to the diagram?* If necessary, probe to see if student can differentiate tens and ones in the diagram and relate these to the tens and ones in the number 67.

Give student the problem 23+15 (in standard vertical form). *Can you explain to me, using the blocks, how to do this problem?*

Prompt student to "think aloud" while doing the problem. Ask if the student has done similar problems in class. Look for examples of metacognitive monitoring. Note mistakes made by the student. Be friendly and give hints only when absolutely necessary. (Make sure any hints and/or prompts are noted below.) When student will continue no farther, ask him (her) if he (she) thinks the answer is correct and why. If the student asks, you may finish discussion of the problem by telling the student whether his (her) answer is correct.

Repeat with the problem 28-15.

8. Mathematics as a "solitary" endeavor

Is math something you can do with other students or do you always have to do it by yourself? If student says you can do it with others, ask when and for what kinds of problems.

If student says he/she works with others at least some of the time, ask *Why do you think your teacher has you do group work in math?*

9. What constitutes mathematics

I am going to show you some questions written on cards. We'll go through the cards one at a time and I'll help you read them if necessary. After you've looked at each card, I'd like you to place it in the **math question** or in the **not a math question** pile and, as you place the card in a pile, tell me why you are putting it in that pile.

Note: Problems will be on separate sheets of paper but interviewer notes should be recorded here.

1. Steve puts out 42 apples. Stacy sells 11. How many are left?

2.
```
  1
  7
+3
```

3. What states are near Indiana?

4. In what year did George Washington become president?

5. Tim and his 2 friends, Sarah and Paul, went fishing. Tim had the longest fishing pole. Sarah's fishing pole was longer than Paul's. Who had the shortest fishing pole?

6. Jerry looked at the toys in the toy store. There were little green turtles, birds, bears, dogs that make noise, and fuzzy cats. Jerry bought 3 different toys. He paid 12 cents in all. Which 3 toys did Jerry buy?

   turtle: 3 cents
   bird: 4 cents
   bear: 8 cents
   dog: 9 cents
   cat: 1 cent

7. Mary saw her friend Jan at the grocery store. Mary said "Hi Jan, how are you?" What do you think Jan said?

8. What kind of pet is most common among students at Dyer? (Interviewer rewords if student doesn't understand question.)

9. How many sides does a square have?

10. What is the next number in the pattern 2, 4, 6, 8, ...?

We are now finished with the interview. Your answers will be very helpful to me and the people from IU that I work with. I enjoyed talking with you. Thanks for your cooperation.

After completion of the interview, on the back of this form the interviewer should write comments about the overall mathematical competence of the student based on responses to questions throughout the interview.

Spring 1990 Dyer Student Interview Schedule:
Grade 4 (or late grade 3)

Interviewer _____ **Student** _____

Time and Date _____

Reminders for interviewers:
Schedule 45 minutes for each interview and allow 15 minutes between interviews to complete your notes. Whenever you prompt a student to get a response, put a "P" before writing that response. In many cases, it will not be possible to write down exactly what a student says. However, whenever you have enough time to include verbatim comments, please do so (put exact wording in quotes). Notes about student non-verbal responses are also appropriate (e.g., nod of the head can be interpreted as a yes or no, confused look should be noted to indicate student didn't understand question, etc.)

Interviewer introduces him/herself and says: *I am interested in your opinions and feelings about some of the kinds of school work you do. The things we talk about will help me understand better*

how you feel about school work. You can tell your parents and teachers about the things we discuss if you want to but I won't be telling them anything that you say. I hope you will be at ease and give me honest responses to the questions I ask. If there are any questions that you don't want to answer, just say you don't want to answer them. There are no right or wrong answers for my questions, I simply want your feelings. OK? Allow child to question you about the interview. If necessary, make additional "small talk" to put child at ease.

1. Background information/mindset of the student

Do you like school? Why or why not?

What is your favorite subject in school? Why?

Are there any subjects you don't like? Why?

If math is not mentioned, ask *Do you enjoy math?* Probe to get an idea of student's general feelings toward math.

How often do you talk with your parents about school? Tell me about the last time you talked with your parents about school.

Do your parents expect you to do well in school? Do they expect you to do well in math?

At home, do you ever watch "Sesame Street" or "Square One," play counting games, card games, talk about numbers, or do other math activities?

2. Usefulness/importance of mathematics

Do you think it is important to learn mathematics? Why? (Probe for both "in school" uses such as "If don't learn it I'll have to go to summer school" and "real world" uses such as "I need to know if I have enough money when I go to the grocery store.")

Do you know anyone who uses math? (If yes) Who? How do they use math?

3. Self confidence

Do you do well in math? Explain.

4. Solving computational and word problems

I would like you to do several math problems for me. First, I would like you to read each problem out loud. I'll help you with the reading if needed. Before you start each problem, I would like you to tell me if you think you can do it. Then, as you work on each problem, I would like you to explain to me exactly what you are thinking and why you are doing everything you do. I want to know everything you think about as you do each problem. When you have finished each problem, I am going to ask you how you felt about doing that problem. OK?

Let student see first problem and help with reading if necessary. Ask the student if he (she) thinks he (she) can do it and why. Then allow student to solve the problem. Prompt student to "think aloud" to see if he (she) relies on the key words and/or any other "cookbook" type of step-by-step solution to the problem. Ask if the student has done similar problems in class.

Look for examples of metacognitive monitoring. Be friendly and give hints only when absolutely necessary. (Make sure any hints and/or prompts are noted below.) When student will continue no farther, ask him (her) if he (she) thinks the answer is correct and why. At this point, show the "continuum" of smile/frown faces and ask the student to select the face that expresses his (her) feelings about the problem. Ask why that face was selected. If the student asks, you may finish discussion of the problem by telling the student whether his (her) answer is correct

Continue with other problems, recording the students' comments and your observations of what the student is doing to solve each problem.

Note: Problems will be on separate sheets of paper but interviewer notes should be recorded here.

1.
```
  463
+209
```

strongly disliked disliked a little uncertain liked a little liked a lot

2. Mary has 6 packages of jelly beans and 4 packages of bubble gum. There are 18 jelly beans in each package. How many jelly beans does she have in all?

strongly disliked disliked a little uncertain liked a little liked a lot

3.
```
  14
x 8
```

strongly disliked disliked a little uncertain liked a little liked a lot

4. There are 6 elephants in the zoo. Together they eat 18 bales of hay each day. If each elephant eats the same amount, how many bales of hay does each one get?

strongly disliked disliked a little uncertain liked a little liked a lot

5. There are 6 elephants in the zoo. Together they eat 18 bales of hay each day. If each elephant eats the same amount, how many bales of hay does each one get?

strongly disliked disliked a little uncertain liked a little liked a lot

5. Importance of word problems

Do you solve word problems in class?

Why do you think your teacher has you solve word problems? (Probe to see if student believes word problems help you to understand math better, if they help you think logically, if

they are good for applying math outside of school, etc.)

6. Task involvement vs. ego involvement ("math mind"/importance of effort)

The point of the following questions is to get at students' opinions on the extent to which anyone can learn mathematics and about whether they judge themselves in relation to other students. Thus, in addition to answers to the questions, note comments about how they know who the best students are (and why) and whether one can be good at mathematics without having to try very hard.

Are there any students who just aren't smart enough to be good at math or can every student learn math if they try hard enough? Don't ask for or accept specific names.

Why do you think there are some students who can't do very well in math? **OR** *How do you know everyone can learn math?*

7. Conceptual understanding/number sense

Tell me what you do during math class.
Probe for comments about manipulatives, amount of time teacher spends explaining new lessons, amount of time spent doing or grading seatwork, etc.

What kinds of things does your teacher do to help you when you're having trouble in math? Look for examples of manipulative or pictorial representations of number and place value, "real life" examples, use of "sample" problems, etc.

If student does not mention manipulatives, point to pattern blocks. *Have you ever seen these before?*

If yes, *Do you use these in class? Explain.*

Do you ever get an answer to a math problem but not really understand if you have done the problem correctly? Probe for additional details, particularly what it means to the child to "understand" mathematics. *How does not understanding a problem make you feel?*

I am going to show you some math materials and pictures. I want you to explain to me what they are and what they mean. (The purpose of these questions is to determine, as much as possible, whether the student can relate abstract number to some concrete or pictorial image and whether the student believes that such representations are useful.)

Show a base-10 block diagram of 357 (from page 34 of the third-grade Harcourt Brace Jovanovich, 1985 text). *Do you know what number this picture represents?*

Get out base-10 blocks. *Have you used these blocks before? Can you explain to me how these blocks relate to the diagram?* If necessary, probe to see if student can differentiate tens and ones in the diagram and relate these to the tens and ones in the number 357.

Give student the problem 326-148 (in standard vertical form). *Can you explain to me, using the blocks, how to do this problem?*

Prompt student to "think aloud" while doing the problem. Ask if the student has done similar problems in class. Look for examples of metacognitive monitoring. Note mistakes made by the student. Be friendly and give hints only when absolutely necessary. (Make sure any hints

and/or prompts are noted below.) When student will continue no farther, ask him (her) if he (she) thinks the answer is correct and why. If the student asks, you may finish discussion of the problem by telling the student whether his (her) answer is correct.

Repeat with the problem 207-188.

8. Mathematics as a "solitary" endeavor

Is math something you can do with other students or do you always have to do it by yourself? If student says you can do it with others, ask when and for what kinds of problems.

If student says he/she works with others at least some of the time, ask *Why do you think your teacher has you do group work in math?*

9. What constitutes mathematics

I am going to show you some questions written on cards. We'll go through the cards one at a time and I'll help you read them if necessary. After you've looked at each card, I'd like you to place it in the **math question** or in the **not a math question** pile and, as you place the card in a pile, tell me why you are putting it in that pile.

Note: Problems will be on separate sheets of paper but interviewer notes should be recorded here.

1. Steve puts out 42 apples. Stacy sells 11. How many are left?

2.
$$\begin{array}{r} 158 \\ 733 \\ +\ 34 \\ \hline \end{array}$$

3. What states are near Indiana?

4. In what year did George Washington become president?

5. Shelly, Maria, Joan, and James each ate a different kind of muffin for breakfast this morning. One ate an apple muffin, one ate a banana muffin, one ate a lemon muffin, and one ate a spice muffin. Maria ate the lemon muffin. James doesn't like banana muffins, and Shelly doesn't like any fruit flavors. Which muffin did each child eat?

6. There were 50 campers at McCormick's Creek when it started to rain. At 6:00 p.m., 5 campers took down their tents and left. At 7:00 p.m., 10 campers took down their tents and left. Each hour, 5 more campers left than at the hour before. At what time did the last campers leave the park?

7. Mary saw her friend Jan at the grocery store. Mary said "Hi Jan, how are you?" What do you think Jan said?

8. What kind of pet is most common among students at Dyer? (Interviewer rewords if student doesn't understand question.)

9. How many sides does a square have?

10. What is the next number in the pattern 1, 4, 7, 10, ...?

We are now finished with the interview. Your answers will be very helpful to me and the people from IU that I work with. I enjoyed talking with you. Thanks for your cooperation.

After completion of the interview, on the back of this form the interviewer should write comments about the overall mathematical competence of the student based on responses to questions throughout the interview.

Spring 1990 Dyer Student Interview Schedule:
Grade 6 (or late grade 5)

Interviewer _____ **Student** _____

Time and Date _____

Reminders for interviewers:
Schedule 45 minutes for each interview and allow 15 minutes between interviews to complete your notes. Whenever you prompt a student to get a response, put a "P" before writing that response. In many cases, it will not be possible to write down exactly what a student says. However, whenever you have enough time to include verbatim comments, please do so (put exact wording in quotes). Notes about student non-verbal responses are also appropriate (e.g., nod of the head can be interpreted as a yes or no, confused look should be noted to indicate student didn't understand question, etc.)

Interviewer introduces him/herself and says: *I am interested in your opinions and feelings about some of the kinds of school work you do. The things we talk about will help me understand better how you feel about school work. You can tell your parents and teachers about the things we discuss if you want to but I won't be telling them anything that you say. I hope you will be at ease and give me honest responses to the questions I ask. If there are any questions that you don't want to answer, just say you don't want to answer them. There are no right or wrong answers for my questions, I simply want your feelings. OK?* Allow child to question you about the interview. If necessary, make additional "small talk" to put child at ease.

1. Background information/mindset of the student

Do you like school? Why or why not?

What is your favorite subject in school? Why?

Are there any subjects you don't like? Why?

If math is not mentioned, ask *Do you enjoy math?* Probe to get an idea of student's general feelings toward math.

How often do you talk with your parents about school? Tell me about the last time you talked with your parents about school.

Do your parents expect you to do well in school? Do they expect you to do well in math?

At home, do you ever watch "Sesame Street" or "Square One," play counting games, card games, talk about numbers, or do other math activities?

2. Usefulness/importance of mathematics

Do you think it is important to learn mathematics? Why? (Probe for both "in school" uses such as "If don't learn it I'll have to go to summer school" and "real world" uses such as "I need to know if I have enough money when I go to the grocery store.")

Do you know anyone who uses math? (If yes) Who? How do they use math?

3. Self confidence

Do you do well in math? Explain.

4. Solving computational and word problems

I would like you to do several math problems for me. First, I would like you to read each problem out loud. I'll help you with the reading if needed. Before you start each problem, I would like you to tell me if you think you can do it. Then, as you work on each problem, I would like you to explain to me exactly what you are thinking and why you are doing everything you do. I want to know everything you think about as you do each problem. When you have finished each problem, I am going to ask you how you felt about doing that problem. OK?

Let student see first problem and help with reading if necessary. Ask the student if he (she) thinks he (she) can do it and why. Then allow student to solve the problem. Prompt student to "think aloud" to see if he (she) relies on the key words and/or any other "cookbook" type of step-by-step solution to the problem. Ask if the student has done similar problems in class. Look for examples of metacognitive monitoring. Be friendly and give hints only when absolutely necessary. (Make sure any hints and/or prompts are noted below.) When student will continue no farther, ask him (her) if he (she) thinks the answer is correct and why. At this point, show the "continuum" of smile/frown faces and ask the student to select the face that expresses his (her) feelings about the problem. Ask why that face was selected. If the student asks, you may finish discussion of the problem by telling the student whether his (her) answer is correct

Continue with other problems, recording the students' comments and your observations of what the student is doing to solve each problem.

Note: Problems will be on separate sheets of paper but interviewer notes should be recorded here.

1.
```
  4.63
+2.09
```

strongly disliked disliked a little uncertain liked a little liked a lot

2. Mary has 16 packages of jelly beans and 24 packages of bubble gum. There are 68 jelly beans in each package. How many jelly beans does she have in all?

strongly disliked disliked a little uncertain liked a little liked a lot

3.
$$\begin{array}{r} 354 \\ \times\ 26 \\ \hline \end{array}$$

strongly disliked disliked a little uncertain liked a little liked a lot

4. There are 256 seats on an airplane. If there are 8 seats in each row, how many rows are there?

strongly disliked disliked a little uncertain liked a little liked a lot

5. Sylvia dropped a tennis ball from a balcony 16 feet above the sidewalk. Each time the ball bounced, it bounced half as high as the previous bounce. Sylvia's brother caught the ball when it bounced exactly 1 foot from the sidewalk. How many times did the ball bounce?

strongly disliked disliked a little uncertain liked a little liked a lot

5. Importance of word problems

Do you solve word problems in class?

Why do you think your teacher has you solve word problems? (Probe to see if student believes word problems help you to understand math better, if they help you think logically, if they are good for applying math outside of school, etc.)

6. Task involvement vs. ego involvement ("math mind"/importance of effort)

The point of the following questions is to get at students' opinions on the extent to which anyone can learn mathematics and about whether they judge themselves in relation to other students. Thus, in addition to answers to the questions, note comments about how they know who the best students are (and why) and whether one can be good at mathematics without having to try very hard.

Are there any students who just aren't smart enough to be good at math or can every student learn math if they try hard enough? Don't ask for or accept specific names.

Why do you think there are some students who can't do very well in math? **OR** *How do you know everyone can learn math?*

7. Conceptual understanding/number sense

Tell me what you do during math class.
Probe for comments about manipulatives, amount of time teacher spends explaining new lessons, amount of time spent doing or grading seatwork, etc.

What kinds of things does your teacher do to help you when you're having trouble in math?
Look for examples of manipulative or pictorial representations of number and place value,

"real life" examples, use of "sample" problems, etc.

If student does not mention manipulatives, point to pattern blocks. *Have you ever seen these before?*

If yes, *Do you use these in class? Explain.*

Do you ever get an answer to a math problem but not really understand if you have done the problem correctly? Probe for additional details, particularly what it means to the child to "understand" mathematics. *How does not understanding a problem make you feel?*

I am going to show you some math materials and pictures. I want you to explain to me what they are and what they mean. (The purpose of these questions is to determine, as much as possible, whether the student can relate abstract number to some concrete or pictorial image and whether the student believes that such representations are useful.)

Show a base-10 block diagram of 1.21 (from practice worksheet 102 of the second-grade Harcourt Brace Jovanovich, 1985 text). *Do you know what number this picture represents?*

Get out pattern blocks. Tell the student that the yellow hexagon represents one. Ask him (her) to find $1/2$ and $1/3$. Give the student the problem $2/3 - 1/2$.

Can you explain to me, using the blocks, how to do this problem?

Prompt student to "think aloud" while doing the problem. Ask if the student has done similar problems in class. Look for examples of metacognitive monitoring. Note mistakes made by the student. Be friendly and give hints only when absolutely necessary. (Make sure any hints and/or prompts are noted below.) When student will continue no farther, ask him (her) if he (she) thinks the answer is correct and why. If the student asks, you may finish discussion of the problem by telling the student whether his (her) answer is correct.

Repeat with the problem $1/3 + 3/4$.

8. Mathematics as a "solitary" endeavor

Is math something you can do with other students or do you always have to do it by yourself? If student says you can do it with others, ask when and for what kinds of problems.

If student says he/she works with others at least some of the time, ask *Why do you think your teacher has you do group work in math?*

9. What constitutes mathematics

I am going to show you some questions written on cards. We'll go through the cards one at a time and I'll help you read them if necessary. After you've looked at each card, I'd like you to place it in the **math question** or in the **not a math question** pile and, as you place the card in a pile, tell me why you are putting it in that pile.

Note: Problems will be on separate sheets of paper but interviewer notes should be recorded here.

1. `Steve puts out 42 apples. Stacy sells 11. How many are left?`

2.
```
  158
  733
+ 34
```

3. What states are near Indiana?

4. In what year did George Washington become president?

5. Shelly, Maria, Joan, and James each ate a different kind of muffin for breakfast this morning. One ate an apple muffin, one ate a banana muffin, one ate a lemon muffin, and one ate a spice muffin. Maria ate the lemon muffin. James doesn't like banana muffins, and Shelly doesn't like any fruit flavors. Which muffin did each child eat?

6. There were 50 campers at McCormick's Creek when it started to rain. At 6:00 p.m., 5 campers took down their tents and left. At 7:00 p.m., 10 campers took down their tents and left. Each hour, 5 more campers left than at the hour before. At what time did the last campers leave the park?

7. Mary saw her friend Jan at the grocery store. Mary said "Hi Jan, how are you?" What do you think Jan said?

8. What kind of pet is most common among students at Dyer? (Interviewer rewords if student doesn't understand question.)

9. How many sides does a square have?

10. What is the next number in the pattern 1, 4, 7, 10, ...?

We are now finished with the interview. Your answers will be very helpful to me and the people from IU that I work with. I enjoyed talking with you. Thanks for your cooperation.

After completion of the interview, on the back of this form the interviewer should write comments about the overall mathematical competence of the student based on responses to questions throughout the interview.

Spring 1991 Highland Park Student Interview Schedule:
All Grades

Interviewer _____ **Student** _____

Grade Level _____ **Student's Teacher** _____

Time and Date _____

Reminders for interviewers:
Interviews will probably take about 15 to 20 minutes each. Whenever you prompt a student to get a response, put a "P" before writing that response. In many cases, it will not be possible to write down exactly what a student says. However, whenever you have enough time to include verbatim comments, please do so (put exact wording in quotes). Notes about student non-verbal responses are also appropriate (e.g., nod of the head can be interpreted as a yes or no, confused look should be noted to indicate student didn't understand question, etc.). Finally, make sure you write summary comments while the interview is fresh in your mind.

Interviewer introduces him/herself and says: **I am interested in your opinions and feelings about some of the kinds of school work you do. The things we talk about will help me understand better how you feel about school work. You can tell your parents and teachers about the things we discuss if you want to but I won't be telling them anything that you say. I hope you will be at ease and give me honest responses to the questions I ask. If there are any questions that you don't want to answer, just say you don't want to answer them. There are no right or wrong answers for my questions, I simply want your feelings. OK?** Allow child to question you about the interview. If necessary, make additional "small talk" to put child at ease.

The first set of questions are designed to build "math autobiographies" of children. Put down answers in narrative form, noting which question a child seems to be addressing. (Ask question #1 and write responses under #1 unless the child is giving information that fits under another number. Then ask question #2, etc.)

Throughout the interview, it is possible that students will have already responded to a specific question. Do not repeat a question that you feel has been answered. (Simply write down "See response to #3" or "already answered".)

1. **To begin, I'd like you to tell me about math. Do you like it? Do you do well in it? Have your ideas about math changed in the last year? Just say anything you'd like about math.** Don't put words in the student's mouth but encourage him or her to talk as much as he or she would like about math.

2. **Tell me whether you've liked math and done well in it in the past. You can talk about math you've done in school or when you've used math at home or at a friend's house. Have you always liked (disliked) math? What makes you like (dislike) math? Were you better at math in the past? Are you doing better now than you have before?** Continue questioning if needed to get at how the child's enjoyment and confidence in math has changed.

3. **Tell me about people who have helped you in math or have made you like (dislike) math? Have your teachers influenced the way you think about math? Have your parents had any affect? Have your friends had any effect?**

4. **Tell me what you do with math at home. Again, you can talk about homework or you can talk about using math that was not part of what you were studying in school. Do your parents help you with math? Do your parents like math? Do your parents expect you to do well in math?**

5. **Are your feelings about school in general the same as those you have about math or do like other subjects better or worse? Have you always felt this way?**

Following are questions which were are similar to those asked last year. Try to get a sense of the student's current feelings and reasons why feelings have changed, particularly in the past year.

6. **Do you think it is important to learn mathematics? Why? Who do you know that uses mathematics?**

7. **Do you think anyone can learn math? Why?**

These questions are new so you don't need to worry about getting comments about how this has changed, although you are certainly welcome to do so.

8. **When do you work hard in math? (Why?) When do you not work very hard in math? (Why?)**

9. **What does it mean to be a good student in math? How do you know who the best students in a class are?**

At this point, thank the student for his or her responses and return the student to the classroom.